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The Role of Fixed Factors in Multi-Sector Neoclassical Growth Models

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The Role of Fixed Factors in Multi-Sector Neoclassical Growth Models

by

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The Role of Fixed Factors in Multi-Sector Neoclassical Growth Models

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My dissertation consists of three essays that examine the role of fixed factors in multi-sector neoclassical growth models, specifically the role of population density in causing the onset of industrialization.

The first paper examines the question of why industrialization occurred first in China rather than England. Although industrialization first occurred in England, it is often thought that China, not England, was the world leader in technology at the time. Yet China did not industrialize until 150 years after England and nearly a century after less advanced European countries. This puzzle is examined in a two-sector model with competing agrarian and industrial production technologies. I find that when differences in population density across countries are accounted for, this delayed industrialization by China is the result of decreasing returns to population density in the agrarian technology and is consistent with the theory.

In the second paper, the importance of total factor productivity (TFP) in causing industrialization is examined. TFP has long been thought to be the driving force behind industrialization. However, such an explanation cannot adequately account for the staggered timing of industrialization across countries. By accounting for differences in population density, a heterogeneity previously unexplored in the literature, I can account for 49-51 percent of the movement toward industrialization in the two sector overlapping generations model employed by Hansen and Prescott (2002).

The third paper presents a sequential competitive equilibrium to solve an infinite horizon two-sector neoclassical growth mode where the two sectors are chosen to represent the agrarian and manufacturing sectors of the economy. In this framework, industrialization is seen to be the relaxing of the non-negativity constrain on the manufacturing sector. It is seen that every country possesses a critical population density upon which it will transition from using solely an agrarian production technology to employing both agrarian and manufacturing technologies. This transition is result of a discrete change in the decision to invest in manufacturing capital. Furthermore, the ability of agents to anticipate industrialization is shown to increase the rate of capital accumulation and hasten the onset of the manufacturing sector.

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Chapter One

Why China Industrialized After England

ABSTRACT

Although industrialization first occurred in England, it is often thought that China, not England, was the world leader in technology at the time. Yet China did not industrialize until 150 years after England and nearly a century after less advanced European countries. This puzzle is examined in a two-sector model with competing agrarian and industrial production technologies. I find that when differences in population density across countries are accounted for, the delayed industrialization of China is the result of decreasing returns to population density in the agrarian technology and consistent with the theory.

Keywords: economic development, population density, industrialization

JEL Codes: N10, N30, O11, O41

Although industrialization first occurred in England, it is often thought that China, not England, was the world leader in technology at the time. Yet China did not industrialize until 150 years after England and nearly a century after less advanced European countries.

Models accounting for this staggered industrialization generally attribute this to significantly lower levels of total factor productivity (TFP) in China, which is at odds with the data as seen in Table 1.1. These different proxies for living standards have been employed as measures of productivity since present-day measures of TFP were not collected in the eighteenth century.

Table 1.1: Measures for TFP, Circa 1750-1800[1]

	Real GNP per Capita	Real Wages of Farm Workers, CPI Deflator	Real Wages of Farm Workers, Calorie Price Deflator	People-Days of Food per Day of Labor[2]	Average Daily Grain Intake (Calories)
England	215	0.011	9961	21.0	2349
China	210	0.009	9996	22.5	2386

While it is impossible to definitively declare a technology leader at the onset of industrialization, these measures have lead economists and historians to conclude that China may have held a slight lead over England at the time.[3]

The reason this is puzzling is that industrialization is generally viewed as a time-dependant event; one in which more technologically advanced countries industrialize at

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[1] Data obtained from Pomeranz (205) (column 1), Bairoch (1981) (column 2), and Allen (2005) (columns 3-5).

[2] Based upon the farming of rice in China and wheat in England.

[3] According to Bairoch (1981): "It is very likely that, in the middle of the eighteenth century, the average standard of living in Europe was a bit lower than that of the rest of the world. This was due to the high level achieved by the Chinese civilization, and the relative importance of that country in the rest of the world (some 36 per cent). Till the years 1815-20 there was still a parity of average income levels between the two regions." Similar statements expressed by Pomeranz (2000), Prescott (1998), Allen (2005), Hansen and Prescott (2002), etc.

an earlier date. Parente and Prescott (2000, 2004) subscribe to this view and explain delays in industrialization by an institutional failure of countries to adopt available technology. Similarly, North (1981) argues that it was more efficient markets and institutions that caused England to industrialize first. However, Keller and Shiue (2004) show that such differences were present only following industrialization.

In an attempt to explain China's failure to industrialize concurrently with England, this paper argues that population density is a more appropriate measure to predict when industrialization should occur. Due to a fixed land supply the agrarian technology exhibits decreasing returns, resulting in a critical population density at which it becomes profitable to begin shifting to a manufacturing technology that is independent of land.

The table below examines a cross section of countries with varying population densities at a given point in time. It shows a positive relationship between population density and the level of industrialization in a country.

Almost three quarters of all countries with a population density below 4 persons per square kilometer have less than 15% of GDP attributable to manufacturing. At the same time, 57% of countries with a population density above 256 persons per square kilometer have more than 25% of GDP attributable to manufacturing.

Table 1.2: Percentage of GDP from Manufacturing[4]

1970 Population Density (persons/sq. km)	Percentage of Countries with <15% of GDP from Manufacturing	Percentage of Countries with 15-25% of GDP from Manufacturing	Percentage of Countries with >25% of GDP from Manufacturing	Total Percentage of Countries
0-4	73%	20%	7%	15
4-16	57%	27%	17%	30
16-64	67%	29%	5%	42
64-256	35%	31%	35%	26
>256	14%	29%	57%	7
All Countries	55%	28%	18%	120

If industrialization is dependent on population density rather than time, then the comparable levels of TFP between England and China should have resulted in industrialization occurring at similar population densities. Table 1.3 confirms that this was the case; the population densities of China and England were almost identical upon their respective industrializations. Further affirming this view, France and Germany, countries with lower levels of TFP at the time, did not industrialize until attaining higher population densities.

Table 1.3: Population Density at Industrialization[5]

Country	Land Mass (million sq. km)	Population Density in 1800 (persons/ sq. km)	Approximate Year of Industrialization	Population Density at Industrialization (persons/ sq. km)
England	0.15	61.67	1800	61.67
China	9.60	34.38	1950	61.46
France	0.55	52.73	1850	65.45
Germany	0.36	50.00	1850	75.00

[4] Data obtained from Boserup (1981) and Developing Countries and Level of Development (1975)
[5] Industrialization determined by initial increase in GDP per capita taken from Table 5.2 in Lucas (2002) and Summers et al. (2002). Population and landmass are obtained by linear interpolation from McEvedy and Jones (1978).

Previously, China’s lag behind England represented a puzzle for the growth literature which predicts that initially more technologically advanced countries will industrialize earlier than less advanced countries. But a theory that accounts for population density would not expect China to industrialize at the same time as England, since in 1800, the time of the British Industrial Revolution, the population density of China was far below that of England. Instead it would accurately predict the date of the Chinese industrial revolution to be a century and a half later when the population density of China reached the level of England circa 1800.

To address this puzzle, I present a two-sector overlapping generations model with agricultural and industrial production technologies identical to those used by Hansen and Prescott (2002). In this model, decreasing returns to population density in the agrarian sector eventually causes the implementation of the industrial production technology. The implications of this model for the timing of industrialization are very accurate.

Chapter two explores alternate explanations for industrialization, like Parente and Prescott (2000, 2004) and Lucas (2002), and compares the relative importance of each. This paper focuses solely on population density to show that when taken into account, the case of China and England does not represent a puzzle, but rather a confirmation of two sector growth models like Hansen and Prescott (2002).

The next section describes the environment while Section III solves the model and shows how the inclusion of population density differences can correct for the late industrialization of China. Section IV concludes and discusses implications of the results.

II. Environment

The economy is identical to the one described by Hansen and Prescott (2002) and Parente and Prescott (2004)[6]. I am presenting it here for completeness and to expand upon the conclusions previously drawn from the model.

There are three factors of production in the economy: land, labor, and capital. There are two types of firms: agrarian firms that have access to a Malthusian production technology, $Y_{a,t} = \gamma_{a,t} K_{a,t}^{\phi} N_{a,t}^{\mu} L_{a,t}^{1-\mu-\phi}$, using capital, labor, and land; and industrial firms that have access to a Solow production technology, $Y_{m,t} = \gamma_{m,t} K_{m,t}^{\theta} N_{m,t}^{1-\theta}$, using capital and labor. Both production technologies produce an identical aggregate good and firms face perfectly competitive markets for labor, land, capital, and goods, such that capital depreciates fully each period. Therefore, firms solve a static profit maximization problem. TFP evolves at the constant, non-stochastic rate $\gamma_{a,t} = \gamma_a^t \gamma_{a,0}$ and $\gamma_{m,t} = \varepsilon_i \gamma_m^t \gamma_{m,0}$, where $\gamma_{j,0}$ is the initial level of TFP in sector $j = a, m$ and $\gamma_a = \gamma_m$ such the growth rate of TFP is identical across sectors prior to industrialization.[7] $\varepsilon_i \in [0,1]$ represents the percentage of world TFP that has been absorbed by the manufacturing sector of country i.

Agents live for two periods and choose labor, investment in capital, and land holdings when young and capital when old to maximize lifetime utility. Preferences are

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[6] The overlapping generations specification of Hansen and Prescott (2002) is chosen to prevent capital accumulation prior to industrialization. An infinitely lived agent model like Parente and Prescott (2004) produces comparable results but lacks the closed form solutions of the simpler framework.
[7] This final specification follows from the calibration in Parente and Prescott (2004)

discounted by $\beta < 1$ such that lifetime utility of a generation t agent is represented by $\ln\left(c_t^t\right) + \beta \ln\left(c_{t+1}^t\right)$.

In period t, there are N_t generation t young agents, each choosing to divide his unit endowment of labor between agrarian labor, $n_{a,t}$, and industrial labor, $n_{m,t}$, in return for wages $\omega_{a,t}$ and $\omega_{m,t}$. The generation t young agent divides his labor income between consumption, c_t, land, $q_t l_{t+1}$, agrarian capital, $k_{a,t+1}$, and industrial capital, $k_{m,t+1}$. The following period, the generation t old agent receives $r_{Ka,t+1} + 1 - \delta$ for each unit of agrarian capital, $r_{Km,t+1} + 1 - \delta$ for each unit of industrial capital, $r_{L,t+1}$ for each unit of land, and sells his land holdings at price q_{t+1} to the $N_{t+1} = \eta N_t$ generation $t+1$ young agents.

III. Equilibrium

The static maximization decision made each period by the representative agrarian firm is

$$\begin{aligned} &\max_{N_{a,t}, K_{a,t}, L_t} \gamma_{a,t} K_{a,t}^{\phi} N_{a,t}^{\mu} L_{a,t}^{1-\mu-\phi} - \omega_{a,t} N_{a,t} - r_{Ka,t} K_{a,t} - r_{L,t} L_t \\ &s.t. \quad N_{a,t}, K_{a,t}, L_t \geq 0 \end{aligned} \tag{1.1}$$

while the decision of the manufacturing firm is

$$\begin{aligned} &\max_{N_{m,t}, K_{m,t}} \gamma_{m,t} K_{m,t}^{\theta} N_{m,t}^{1-\theta} - \omega_{m,t} N_{m,t} - r_{Km,t} K_{m,t} \\ &s.t. \quad N_{m,t}, K_{m,t} \geq 0 \end{aligned} \tag{1.2}$$

By taking the first order condition with respect to land, one can see that as $N_{a,t}$ or $K_{a,t}$ approach zero, $r_{L,t}$ approaches zero as well. Assuming firms employ factors when indifferent, this zero cost of land in the limit implies that the agrarian sector will always use all available land. However, so long as $L>0$, the marginal return to labor and capital as $\{N_{a,t},K_{a,t}\}\to 0$ will be infinite, so it must be that $N_{a,t},K_{a,t}>0$ and the agrarian firm always produces.

Conversely, substituting the first order conditions with respect to manufacturing capital into the argument of the maximization function, we find that the industrial sector will only yield non-negative profits, and hence operate, when[8]

$$\gamma_{m,t} > \left(\tfrac{r_t}{\theta}\right)^{\theta}\left(\tfrac{\omega_t}{1-\theta}\right)^{1-\theta} \tag{1.3}$$

Equation (1.3) is the result of Proposition 2, equation (8), of Hansen and Prescott (2002). They found a condition on manufacturing TFP, relative to factor prices, upon which an economy will industrialize. The contribution of this paper is in showing that factor prices are functions of population density and by substituting for factor prices in (1.3), it is possible to derive conditions for industrialization on a more fundamental and easily measurable state variable: population density. This revised condition allows the staggered timing of industrialization across countries to be explained by differences in population densities, a source of heterogeneity not accounted for by Hansen and Prescott (2002) or Parente and Prescott (2004).

For times when (1.3) does not hold, only the agrarian sector is employed, and the wage and rental price of capital will be set by the agrarian firm's problem, where $N_{a,t}=N_t$ and $K_{a,t}=K_t$. Substituting for $\omega_{a,t}$ and $r_{Ka,t}$, and expressing capital as k_tN_t,

[8] The proof is omitted here since the results follow directly from Hansen and Prescott (2002).

where k_t is capital per capita, we find that the industrial technology is employed if and only if population density is above the critical level,

$$\left(\frac{N_t}{L}\right)^* \geq \left[\left(\frac{\gamma_{a,t}}{\gamma_{m,t}}\right)\left(\frac{\phi}{\theta}\right)^{\theta}\left(\frac{\mu}{1-\theta}\right)^{1-\theta}\left(\frac{1}{k_t}\right)^{\theta-\phi}\right]^{1/1-(\mu+\phi)} \tag{1.4}$$

To remove capital per capita from (1.4), it is necessary to address the agent's problem. The utility maximization problem for a generation t agent is

$$\begin{aligned} \max_{k_{t+1}, l_{t+1}, c_t^t, c_{t+1}^t} \quad & \ln\left(c_t^t\right) + \beta \ln\left(c_{t+1}^t\right) \\ s.t. \quad & c_t^t + k_{t+1} + q_t l_{t+1} = \omega_t \\ & c_{t+1}^t = r_{K,t+1} k_{t+1} + \left(r_{L,t+1} + q_{t+1}\right) l_{t+1} \end{aligned} \tag{1.5}$$

It is not difficult to prove that when the growth rate of agrarian TFP is $\gamma_a = \eta^{1-(\mu+\phi)}$, there exists an equilibrium in which capital, consumption and output per worker are constant.

In this balanced growth path, the path to industrialization can be completely characterized by two variables, $\left(\frac{\gamma_a}{\gamma_m}\right)$ and $\left(\frac{N}{L}\right)$, such that the condition for industrialization can be rewritten as[9]

$$\left(\frac{N}{L}\right) \geq \left(\frac{\gamma_a}{\gamma_m}\right)^{1/1-(\mu+\phi)}\left[\left(\frac{\gamma_{a,0}}{\gamma_{m,0}}\right)\left(\frac{\phi}{\theta}\right)^{\theta}\left(\frac{\mu}{1-\theta}\right)^{1-\theta}\left(\frac{1}{k_a}\right)^{\theta-\phi}\right]^{1/1-(\mu+\phi)} \tag{1.6}$$

To compare the implications of the model with and without accounting for differences in population density, I reconstruct the work of Parente and Prescott (2004). They argue that all countries industrialize at the same level of manufacturing TFP. To generate the differences in manufacturing TFP necessary to account for the staggered

[9] In chapter two, the full general equilibrium is solved to reduce k_a to fundamental parameters. Since the value of k_a is not needed for the results in this paper, its value is not explored further here.

timing of industrialization, they introduce a country specific variable, $\varepsilon_i \in [0,1]$, that represents the percentage of world TFP that has been absorbed by the manufacturing sector of country i. Equation (1.6) is modified by substituting in the TFP paths, rewriting the population density at time t as $\frac{N}{L} = \eta^t \frac{N_0}{L}$, and setting $\gamma_a = \gamma_m$. After rearranging, the result is

$$\varepsilon \gamma_{m,0} \eta^{[1-(\mu+\phi)]t} \geq \left[\left(\gamma_{a,0}\right) \left(\frac{\phi}{\theta}\right)^{\theta} \left(\frac{\mu}{1-\theta}\right)^{1-\theta} \left(\frac{1}{k_{ss}}\right)^{\theta-\phi} \left(\frac{N_0}{L}\right)^{(\mu+\phi)-1} \right] \tag{1.7}$$

where the right hand side is assumed to be identical across countries. The result is a world in which all countries industrialize at the same level of industrial TFP and ε_i is calibrated to account for differences in the timing. Equation (1.8) corresponds to (4.2) of Parente and Prescott (2004).

$$\varepsilon_i \eta^{[1-(\mu+\phi)]\tau_i} = \varepsilon_j \eta^{[1-(\mu+\phi)]\tau_j} \tag{1.8}$$

where τ_i is the year of industrialization of country i.

Allowing $\frac{N_0}{L}$, the initial population density, to vary across countries, causes (1.7) to instead imply equality of population density at industrialization, after accounting for differences in TFP absorption.[10]

$$\varepsilon_i^{\frac{1}{1-(\mu+\phi)}} \left(\frac{N_{0,i}}{L_i}\right) \eta_i^{\tau_i} = \varepsilon_j^{\frac{1}{1-(\mu+\phi)}} \left(\frac{N_{0,j}}{L_j}\right) \eta_j^{\tau_j} \tag{1.9}$$

[10] Without any loss of generality, $\left(\frac{N_{0,i}}{L_i}\right)$ will be chosen as country i's population density in 1800.

$\frac{N_{t,i}}{L_i} = \frac{N_{0,i}}{L_i}\eta_i^t$ is the population density of country i at time t. Equation (1.9) implies that industrialization should occur at a given population density after adjusting for TFP adoption rates. The parameter ε_i is calibrated to account for such deviations.

Using equations (1.8) and (1.9), respectively, we can determine the relative TFP adoption rates of China and England that are needed to replicate the timing of industrialization when neglecting

$$\frac{\varepsilon_{england}}{\varepsilon_{china}} = \left(\eta_{china}^{\left(\tau_{china} - \tau_{england}\right)}\right)^{1-(\mu+\phi)} \tag{1.10}$$

and accounting for differences in population density.

$$\frac{\varepsilon_{england}}{\varepsilon_{china}} = \left(\frac{\left(\frac{N_{\tau_C}}{L}\right)_{china}}{\left(\frac{N_{\tau_e}}{L}\right)_{england}}\right)^{1-(\mu+\phi)} \tag{1.11}$$

The year of industrialization, τ_i, and the population density at industrialization, $\frac{N_{\tau_i}}{L}$, of each country follows from Table 1.3, the exogenous parameters $\mu = .6$ and $\phi = .1$ are taken from Parente and Prescott (2004), and the population growth rate of China between 1800 and 1950, $\eta_{china} = 1.005$, was derived from McEvedy and Jones (1978).

As seen in Table 1.4, taking differences in population density into account reverses the implied technology lag of China relative to England from one that is at odds with the data to one that is consistent with observations of comparable TFP.

Table 1.4: Relative TFP Implied by Timing of Industrialization

Country	Implied Efficiency Ignoring Population Density Differences [Implied ε_i from (1.8) and (1.10)]	Implied Efficiency Accounting for Population Density Differences [Implied ε_i from (1.9) and (1.11)]
England	1.00	.998
China	.793	1.00

IV. Conclusion

Ignoring differences in population density across countries leads to the conclusion that there is a critical level of TFP at which every country industrializes. However, this results in a counterfactual when comparing China and England.

By accounting for differences in population density, this case no longer represents a contradiction between theory and data. This paper shows that once China's initial lower population density is accounted for, the existing growth theory accurately predicts the timing of its industrialization. It is seen is that China did not industrialize concurrently with England because it had a significantly lower level of population density in 1800. Only once its population density increased, making land a sufficiently scarce resource, did manufacturing become a profitable alternative to agriculture in China. It was at this point that China industrialized.

The use of population density to calibrate the relative absorption of TFP across countries should improve the accuracy of previous estimates. Using these results in a manner similar to Parente and Prescott (2004) would allow for additional heterogeneity between countries, perhaps providing for new explanations of the differing growth rates following industrialization. It may be that post-industrial growth paths depend upon the ratio of TFP to population density at industrialization. Such lines of research may ultimately lend insight into the cause of Asian growth miracles of the 1950's.

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Chapter Two

Dispelling the Myth that TFP is Critical

ABSTRACT

Total factor productivity (TFP) has long been thought to be the driving force behind industrialization. However, such an explanation cannot adequately account for the staggered timing of industrialization across countries. By accounting for differences in population density, a heterogeneity previously unexplored in the literature, I can account for 49-51 percent of the movement toward industrialization in the two sector overlapping generations model employed by Hansen and Prescott (2002).

Keywords: economic development, population density, TFP, industrialization

JEL Codes: N10, N30, O11, O41

Total factor productivity (TFP) has long been thought to be the driving force behind industrialization. However, such an explanation cannot adequately account for the staggered timing of industrialization across countries. In this paper, I explore the role of population density in the two-sector overlapping generations unified growth theory of Hansen and Prescott (2002) and prove that it is capable of adding the additional heterogeneity necessary to match the timing of industrialization.

Using an overlapping generations model, I derive a closed form solution for the critical population density at which an economy using solely an agrarian production technology to one employing both agrarian and manufacturing technologies. For the purposes of this paper, industrialization is defined as this transition. Since the critical population density is a function of TFP, movement towards industrialization can be broken down into two effects: a TFP effect and a population density effect. The relative magnitudes of these effects are then compared and the population density effect is found to explain 50% of industrialization.

The role of population density in industrialization is not novel. Bosserup (1981) argued that population density was the cause behind technology growth and more recently, Guillo and Sebastian (2004) explored the role that fixed natural resources have on a country's development. However, there is no literature examining the role of population density on industrialization, treating TFP and population density independently. While this independence does not actually exist, it is still useful to explore the different pathways through which variables effect industrialization.

The next section describes the environment. Section III removes TFP growth and capital from the model and solves this elementary problem to provide intuition. The full model is returned to in Section IV where an equilibrium path is found and the results from the previous section are shown to be robust to the full specifications. Section V takes the model to the data so that the population density and TFP effects can be compared. Section VI concludes.

II. <u>Environment</u>

The economy is similar to the overlapping generations model of Hansen and Prescott (2002). However, the TFP path is changed to allow for interactions between countries.

There are three resources in the economy: land, labor, and capital. There are two types of firms: agrarian firms that have access to a Malthusian production technology, $Y_{a,t} = \gamma_{a,t} K_{a,t}^{\phi} N_{a,t}^{\mu} L_{a,t}^{1-\mu-\phi}$, that uses capital, labor, and land, and industrial firms that have access to a Solow production technology, $Y_{m,t} = \gamma_{m,t} K_{m,t}^{\theta} N_{m,t}^{1-\theta}$, that utilizes only capital and labor. Both production technologies produce an identical aggregate good and all firms face perfectly competitive markets for labor, land, and capital, where capital depreciates fully each period. Each sector can be reduced to a single representative firm. There are no fixed costs and no savings technology is available to firms. Therefore, firms solve a static maximization problem and operate only in periods where profits are nonnegative.

TFP in sector $i = a, m$ evolves according to

$$\gamma_{i,t+1}^{j}=\left(\xi_{a}\left(\frac{N_{a,t}^{j}}{N_{t}^{j}}\right)+\xi_{m}\left(\frac{N_{t}^{j}-N_{a,t}^{j}}{N_{t}^{j}}\right)\right)\cdot\sigma_{i,t}^{j}\cdot\gamma_{i,t}^{j} \tag{2.1}$$

where $\sigma_{i,t}^{j}\geq 1$, represents the absorption of technology from abroad for country j in sector i for period t, and $\xi_a=\eta^{1-(\mu+\phi)}$ and $\xi_m=1.012$ are chosen to match the growth paths of Malthusian and Solow economies, respectively. Furthermore it is assumed that initially technology is equal across countries, so $\sigma_{i,t}^{t}=1$ prior to the industrialization of the first country.

Agents live for two periods and choose the distribution of labor, investment in capital, and land holdings when young and distribution of capital when old to maximize lifetime utility. Period utility is discounted in time by $\beta<1$ such that lifetime utility of a generation t agent is represented by $\ln\left(c_t^t\right)+\beta\ln\left(c_{t+1}^t\right)$. In period t, there are N_t generation t young agents, each choosing to divide his unit endowment of labor between agrarian labor, $n_{a,t}$, and industrial labor, $n_{m,t}$, in return for wages $\omega_{a,t}$ and $\omega_{m,t}$. The generation t young agent then divides his labor income between consumption, c_t, land investment, $q_t l_{t+1}$, agrarian capital, $k_{a,t+1}$, and industrial capital, $k_{m,t+1}$. The following period, the generation t old agent receives $r_{Ka,t+1}$ for each unit of agrarian capital, $r_{Km,t+1}$ for each unit of industrial capital, $r_{L,t+1}$ for each unit of land, and sells his land holdings at price q_{t+1} to the $N_{t+1}=\eta N_t$ generation $t+1$ young agents.

To gain an intuitive understanding of how changing factor endowments are capable of generating the transition from an agrarian to an industrialized economy, I first show an example without TFP growth or capital. These results are then generalized.

III. Simple Example

In this simplified version of the model there is no TFP growth, $\gamma_a = \gamma_m = 1$, and no capital, $\phi = \theta = 0$. The agrarian firm's maximization problem becomes

$$\begin{aligned} &\max_{N_{a,t}, L_t} N_{a,t}^{\mu} L_t^{1-\mu} - \omega_{a,t} N_{m,t} - r_{L,t} L_t \\ &s.t. \quad N_{a,t} \geq 0 \\ &\qquad\quad L_t \geq 0 \end{aligned} \tag{2.2}$$

and the industrial firm's problem becomes

$$\begin{aligned} &\max_{N_{m,t}} N_{m,t} - \omega_{m,t} N_{m,t} \\ &s.t. \quad N_{m,t} \geq 0 \end{aligned} \tag{2.3}$$

What is important to note is that the agrarian technology exhibits decreasing returns to scale with respect to labor.

Since agents are indifferent between supplying labor to the agrarian or manufacturing firm, it is not profitable for the industrial firm to produce when the $\omega_t \geq 1$. In such cases only the agrarian firm produces and pays workers and land owners the marginal product of labor and land, respectively, with $\frac{N_{a,t}}{L} = \frac{N_t}{L}$. However, once $\frac{N_t}{L}$ becomes sufficiently large, this wage is reduced to one and both technologies are employed. Since the industrial firm always pays wages $\omega_{m,t} = 1$, the agrarian firm can never pay a lower wage.

The population density upon which the industrial technology is first employed, $\left(\frac{N}{L}\right)^* = \mu^{1/(1-\mu)}$, will be referred to as the critical population density. After the critical population density is reached, all additional workers are employed in the manufacturing sector.

$$N_{a,t} = \begin{cases} N_t & 0 \le \frac{N_t}{L} \le \mu^{\frac{1}{1-\mu}} \\ \mu^{\frac{1}{1-\mu}} L & \frac{N_t}{L} > \mu^{\frac{1}{1-\mu}} \end{cases} \tag{2.4}$$

Accordingly, wages are falling due to the decreasing returns to population density in the agrarian sector and only stabilize with the implementation of the industrial sector.

$$\omega_t = \begin{cases} \mu\left(\frac{N_t}{L}\right)^{\mu-1} & 0 \le \frac{N_t}{L} \le \mu^{\frac{1}{1-\mu}} \\ 1 & \frac{N_t}{L} > \mu^{\frac{1}{1-\mu}} \end{cases} \tag{2.5}$$

Conversely, the rental rate of land, $r_{L,t}$, increases with population density and only stabilizes when labor is diverted to the manufacturing sector.

$$r_{L,t} = \begin{cases} (1-\mu)\left(\frac{N_t}{L}\right)^{\mu} & 0 \le \frac{N_t}{L} \le \mu^{\frac{1}{1-\mu}} \\ (1-\mu)\mu^{\frac{1}{1-\mu}} L & \frac{N_t}{L} > \mu^{\frac{1}{1-\mu}} \end{cases} \tag{2.6}$$

Output per capita follows wages in decreasing in density before being stabilized by the implementation of the manufacturing technology.

$$\frac{Y_t}{N_t} = \begin{cases} \left(\frac{N_t}{L}\right)^{\mu-1} & N_t \le \mu^{\frac{1}{1-\mu}} \\ 1+(1-\mu)\dfrac{\mu^{\frac{\mu}{1-\mu}}}{(N_t/L)} & N_t \ge \mu^{\frac{1}{1-\mu}} \end{cases} \tag{2.7}$$

The transition of these four variables upon industrialization resemble the data and the main discrepancy, the roughly $\frac{\pi}{4}$ shift of wage and output per capita will be corrected in the next section with the addition of capital and TFP growth.

In the jargon of classical trade theory, the critical population density represents the outer boundary of the diversification cone. Rybcyznski (1955) states that the increase of a factor endowment will result in a relative increase in output for the technologies that are more intensive in that factor. Figure 2.1 graphically depicts this for the simple model.

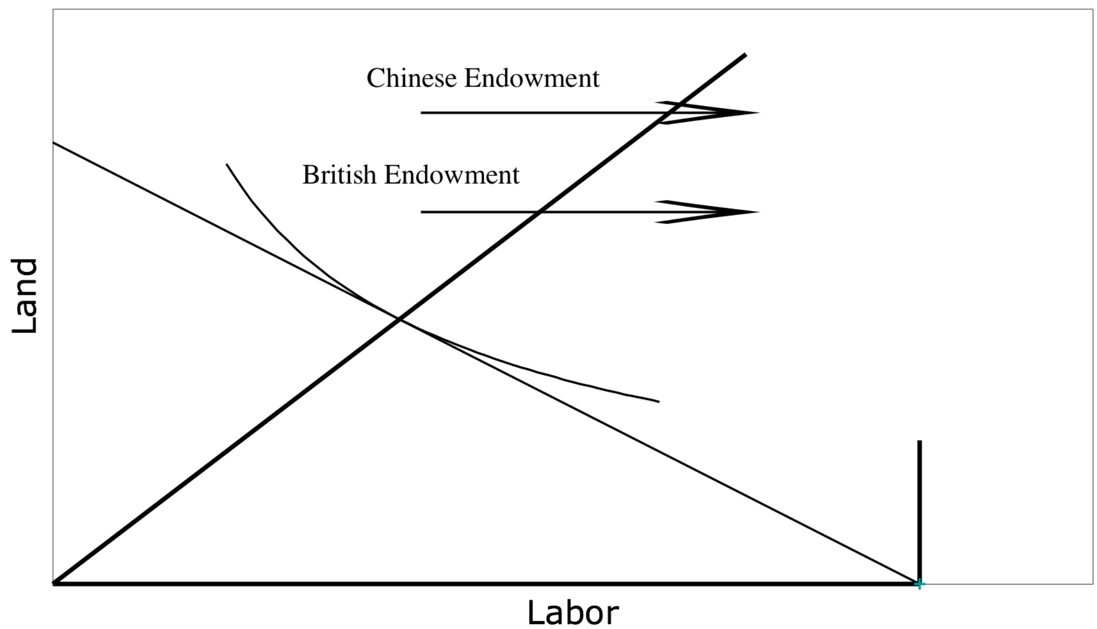
In Figure 2.1, the unit output isoquants are plotted for both sectors. The line tangent to both isoquants is the factor price equalization (FPE) line, the slope of which is the absolute value of the ratio of factor prices when both technologies are in use. Because it is possible to employ both technologies profitably only when the endowment of the economy is within the cone formed by the tangency points of the unit cost isoquants and the FPE line, Lerner (1952) refers to this space as the diversification cone.

For any factor endowment inside this the cone, the maximum level of output is achieved by use of both technologies. For points outside the diversification cone, the maximum level of output is achieved by using only one technology; in such cases FPE need not hold.

Increasing population density causes the economy to enter the diversification cone. Once inside the diversification cone, both sectors are employed and FPE holds. Industrialization can thus be seen as the movement of a country's factor endowments into the diversification cone. This transition is demonstrated on the diagram from two initial endowments, chosen to represent China and England. A higher initial population density results in England entering the diversification cone prior to China.

Another way this transition can occur is if the diversification cone widens. This occurs when manufacturing TFP increases relative to agrarian TFP. Both causes are seen in the data, and Section V compares these two effects.

Figure 2.1: Lerner-Pearce Diagram



Regardless of the cause, the abundance of land relative to labor makes the agrarian technology more efficient prior to attaining the critical population density. However, once population density is sufficiently high, there exists an optimal mix of the two production technologies and both sectors are employed.

IV. <u>Equilibrium</u>

Capital and TFP are now added so that the static maximization problem of the representative agrarian firm becomes

$$\begin{aligned} &\max_{N_{a,t},K_{a,t},L_t} \gamma_{a,t} K_{a,t}^{\phi} N_{a,t}^{\mu} L_{a,t}^{1-\mu-\phi} - \omega_{a,t} N_{a,t} - r_{Ka,t} K_{a,t} - r_{L,t} L_t \\ &s.t. \quad N_{a,t}, K_{a,t}, L_t \geq 0 \end{aligned} \tag{2.8}$$

while the problem of the industrial firm becomes

$$\begin{aligned} &\max_{N_{m,t},K_{m,t}} \gamma_{m,t} K_{m,t}^{\theta} N_{m,t}^{1-\theta} - \omega_{m,t} N_{m,t} - r_{Km,t} K_{m,t} \\ &s.t. \quad N_{m,t}, K_{m,t} \geq 0 \end{aligned} \tag{2.9}$$

I require that that $\theta/(1-\theta) > \phi/\mu$ and $\theta > \phi$ to ensure that the industrial technology is more intensive in capital than labor and that its capital to labor intensity ratio is higher than that of the agrarian technology.

By taking the first order condition with respect to land, one can see that as $N_{a,t}$ or $K_{a,t}$ approach zero, $r_{L,t}$ approaches zero as well. Assuming firms employ factors when indifferent, this zero cost of land in the limit implies that the agrarian sector will always use all available land. However, so long as $L > 0$, the marginal return to labor and capital as $\{N_{a,t}, K_{a,t}\} \to 0$ will be infinite, so it must be that $N_{a,t}, K_{a,t} > 0$ and the agrarian firm always produces.

Conversely, substituting the first order conditions with respect to manufacturing capital into the argument of the maximization function, we find that the industrial sector will only yield non-negative profits, and hence operate, when[11]

$$\gamma_{m,t} > \left(\tfrac{r_t}{\theta}\right)^{\theta} \left(\tfrac{\omega_t}{1-\theta}\right)^{1-\theta} \tag{2.10}$$

Since this is a static decision, failure to achieve positive profits will result in a shutdown of the sector.

[11] The proof is omitted here since the results follow directly from Hansen and Prescott (2002).

Up to now, all the results have followed directly from Hansen and Prescott (2002). The new contribution of this paper begins here with the removal of factor prices from (1.3). For times when (1.3) does not hold, only the agrarian sector is employed, and the wage and rental price of capital will be set by the agrarian firm where $N_{a,t} = N_t$ and $K_{a,t} = K_t$. Substituting in $\omega_{a,t}$ and $r_{Ka,t}$, and recognizing that for any given period capital can be expressed as $k_t N_t$, where k_t is capital per capita, we find that the industrial technology is employed if and only if population density is above the critical level,

$$\left(\frac{N_t}{L}\right)^* \geq \left[\left(\frac{\gamma_{a,t}}{\gamma_{m,t}}\right)\left(\frac{\phi}{\theta}\right)^{\theta}\left(\frac{\mu}{1-\theta}\right)^{1-\theta}\left(\frac{1}{k_t}\right)^{\theta-\phi}\right]^{1/1-(\mu+\phi)} \tag{2.11}$$

The maximization problem for the generation t agent can be described by

$$\begin{aligned} U_t = &\max_{k_{s,t+1},l_{t+1},k_{m,t+1},n_{m,t},n_{s,t},c_t^t,c_{t+1}^t} \ln\left(c_t^t\right) + \beta \ln\left(c_{t+1}^t\right) \\ &s.t. \quad c_t^t + k_{a,t+1} + k_{m,t+1} + q_t l_{t+1} = \omega_{a,t} n_{a,t} + \omega_{m,t} n_{m,t} \\ &\qquad c_{t+1}^t = r_{Ka,t+1} k_{a,t+1} + r_{Km,t+1} k_{m,t+1} + \left(r_{L,t+1} + q_{t+1}\right) l_{t+1} \\ &\qquad 0 \leq k_{a,t+1}, k_{m,t+1}, n_{a,t}, n_{m,t} \leq 1 \\ &\qquad n_{a,t} + n_{m,t} = 1 \end{aligned} \tag{2.12}$$

As discussed in the previous section, agents will only supply labor and capital to the highest bidder, so perfect competition amongst firms will equalize factor prices and the agent problem simplifies to

$$\begin{aligned} U_t = &\max_{k_{t+1},l_{t+1},c_t^t,c_{t+1}^t} \ln\left(c_t^t\right) + \beta \ln\left(c_{t+1}^t\right) \\ &s.t. \quad c_t^t + k_{t+1} + q_t l_{t+1} = \omega_t \\ &\qquad c_{t+1}^t = r_{K,t+1} k_{t+1} + \left(r_{L,t+1} + q_{t+1}\right) l_{t+1} \end{aligned} \tag{2.13}$$

After some manipulation, the first order conditions for this simplified problem are

$$q_{t+1} = q_t r_{K,t+1} - r_{L,t+1} \tag{2.14}$$

$$k_{t+1} + q_t l_{t+1} = \frac{\beta}{1+\beta}\omega_t \tag{2.15}$$

Given initial conditions, $\{q_0, k_{a,0}, k_{m,0}, l_0\}$, and an aggregate sequence for population and land, $\{N_t, L_t\}_{t=0}^{\infty}$, an equilibrium growth path can be defined by a sequence of prices, $\{q_t, \omega_t, r_{K,t}, r_{L,t}\}_{t=0}^{\infty}$ and labor allocation, $\{n_{a,t}\}_{t=0}^{\infty}$, and investment, $\{k_{a,t+1}, k_{m,t+1}, l_{t+1}\}_{t=0}^{\infty}$, decisions such that:

When (2.11) does not hold in period $t+1$, generation t young agents with perfect foresight, given prices and allocation in period t, choose $n_{a,t} = 1$, $k_{m,t+1} = 0$, and $\{k_{a,t+1}, l_{t+1}\}$ that satisfy (2.15) and market clearing:

$$N_{t-1}c_t^{t-1} + N_t\left(c_t^t + k_{t+1}\right) = \gamma_{a,t}K_{a,t}^{\phi}N_{a,t}^{\mu}L_{a,t}^{1-\mu-\phi} + \gamma_{m,t}K_{m,t}^{\theta}N_{m,t}^{1-\theta} \tag{2.16}$$

And when (2.11) holds for period $t+1$, agents chose $\{n_{a,t}, k_{a,t+1}, k_{m,t+1}, l_{t+1}\}$, given prices, such that the returns to labor,

$$(1-\theta)\gamma_{m,t+1}K_{m,t+1}^{\theta}N_{m,t+1}^{-\theta} = \mu\gamma_{a,t+1}K_{a,t+1}^{\phi}N_{a,t+1}^{\mu-1}L_{a,t+1}^{1-\mu-\phi} \tag{2.17}$$

and capital,

$$\theta\gamma_{m,t+1}K_{m,t+1}^{\theta-1}N_{m,t+1}^{1-\theta} = \phi\gamma_{a,t+1}K_{a,t+1}^{\phi-1}N_{a,t+1}^{\mu}L_{a,t+1}^{1-\mu-\phi} \tag{2.18}$$

are equalized across sectors, (2.15) holds, and markets clear.

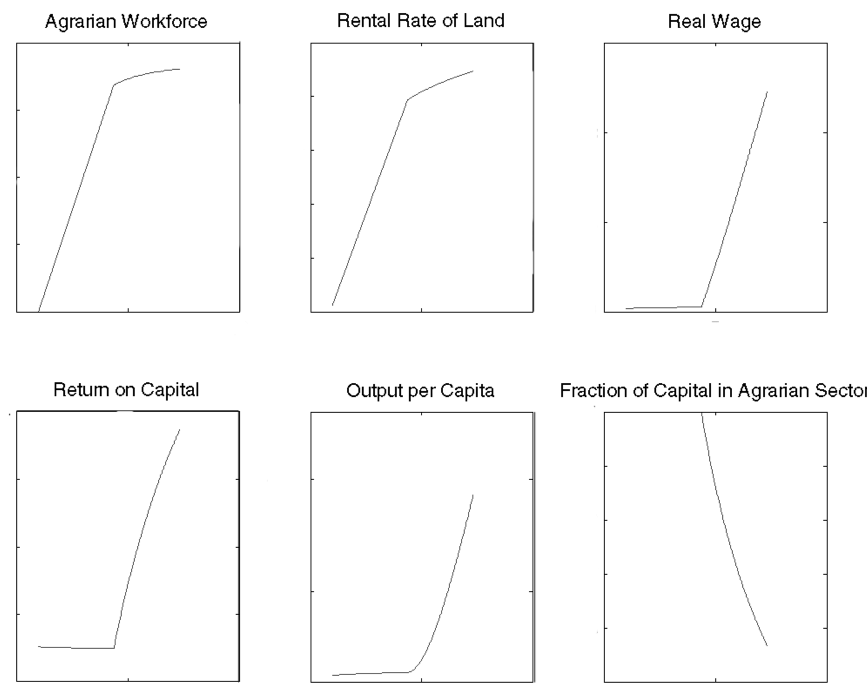
In the special case being examined, where the supply of land is fixed and there are constant population and TFP growth rates while only the agrarian technology is employed; there is a growth path where capital, consumption, and output per worker are constant when $\left(\frac{N_{t+1}}{L}\right) \leq \left(\frac{N_{t+1}}{L}\right)^*$ and then increasing thereafter. A proof of this is included as a supplemental proposition. In the limit as $\left(\frac{N_t}{L}\right) \to \infty$, the economy approaches a balanced growth path where capital, consumption, and output per worker grow at a rate of 2% annually.[12] The TFP growth path, (2.1) was chosen to match these end behaviors while being consistent with the empirical findings of Bairoch (1982) and the theoretical results of Lucas (2002) that TFP should be equal across sectors, but increase following industrialization.

For an economy on this growth path, the path to industrialization can be completely characterized by two variables, $\left(\frac{\gamma_{a,t}}{\gamma_{m,t}}\right)$ and $\left(\frac{N_t}{L}\right)$.

In Figure 2.2, the key economic variables are plotted against population density with TFP equalized across sectors and the remaining exogenous parameter values following from Parente and Prescott (2004), assuming 35 year periods: $\theta = .4$, $\phi = .1$, $\theta = .1$, $\mu = .6$, $\beta = .34$.[13] $\beta = (.97)^{35}$. Notice that the findings from the simple economy presented in Section III hold with the added benefit that trends in real wage and output per capita have rotated so they match the data. Increases to the level of industrial TFP growth have the effect of increasing the rate of per capita output, wages, and rental rate of capital after industrialization as well as lowering the population density necessary for industrialization. Similarly, decreases in industrial TFP growth flatten the graphs and increase the critical population density necessary to achieve industrialization.

[12] This is equivalent to 200% for each 35 year generation.
[13] $\beta = .34$ follows from an annual discount rate of .97.

Figure 2.2: Key Variables Plotted Against Population Density



Conditions that guarantee the occurrence of industrialization are included as a supplemental proposition. So long as industrialization does occur, the critical population density is unique. Rewriting the assumption of constant population growth as $t = \frac{\ln(N/L) - \ln(N_0/L)}{\ln(\eta)}$, time dependence can be removed from the problem and the time independent critical population density for an economy where $\sigma_i = 1$, to be referred to as a closed economy, is

$$\frac{N}{L}^* \geq \left[\left(\frac{\gamma_{a,0}}{\gamma_{m,0}} \right) \left(\frac{\phi}{\theta} \right)^{\theta} \left(\frac{\mu}{1-\theta} \right)^{1-\theta} \left(\frac{1}{k_a} \right)^{\theta-\phi} \right]^{\frac{1}{(1-(\mu+\phi))}} \tag{2.19}$$

To this point, the capability of TFP to evolve at different rates across countries and sectors has not been explored. Under such assumptions $\frac{N}{L}^*$ is identical across countries and equal to the population density of England at industrialization. This is constent with the calibration of Parente and Prescott (2004) that prior to industrialization, $\gamma_{a,i,t} = \gamma_{m,i,t}$. In order to generate the disparity in TFP that Parente and Prescott (2004) use to explain the staggered timing of industrialization (TFP effect), I allow the TFP growth rate to vary for all countries after 1800.

In order to do so, it is necessary to make several other, less restrictive, assumptions. The first assumption is that the TFP growth rate and population density are the only sources of heterogeneity between countries. This implies that $\left[\left(\frac{\gamma_{a,o}}{\gamma_{m,0}}\right)\left(\frac{\phi}{\theta}\right)^{\theta}\left(\frac{\mu}{1-\theta}\right)^{1-\theta}\left(\frac{1}{k}\right)^{\theta-\phi}\right]^{1/1-(\mu+\phi)}$ is constant across countries prior to industrialization. The next assumption is that $\sigma_{i,t<1800} = 1$, while $\sigma_{i,t>1800} \geq 1$. Therefore, prior to 1800, $\left(\frac{\gamma_{a,i}}{\gamma_{m,i}}\right) = 1$ for all countries and $\left[\left(\frac{\gamma_{a,o}}{\gamma_{m,0}}\right)\left(\frac{\phi}{\theta}\right)^{\theta}\left(\frac{\mu}{1-\theta}\right)^{1-\theta}\left(\frac{1}{k}\right)^{\theta-\phi}\right]^{1/1-(\mu+\phi)} = 61.67$, the density at which England industrialized.

Contradictions to this last assumption exist, as England was not the densest country in the world in 1800. More dense countries, such as Belgium, the Netherlands, Italy, and Japan cannot be examined using this analysis and provide evidence of additional heterogeneities that need to be accounted for to truly explain the timing of industrialization.

These assumptions follow from the specification for TFP in Section II. Given that, with the exception of population density, countries begin identically, it is not until England industrializes that additional heterogeneity can occur. When England, and subsequently other countries, industrialize, TFP in those countries increases above the path $\gamma_{i,t} = \xi_a^t \gamma_{i,0}$ and becomes an opportunity for international technology dissipation, represented by $\sigma_{i,t} \geq 1$. Since, TFP varies across sectors due solely to international dispersion, I refer to this as the open economy effect.

When an economy is opened in this manner, the time-independent critical population density becomes

$$\frac{N}{L}^* = \left[\left(\frac{N_0}{L}\right)^{\ln\left(\frac{\sigma_a}{\sigma_m}\right)} \left(\left(\frac{\gamma_{a,0}}{\gamma_{m,0}}\right) \left(\frac{\phi}{\theta}\right)^{\theta} \left(\frac{\mu}{1-\theta}\right)^{1-\theta} \left(\frac{1}{k_a}\right)^{\theta-\phi} \right)^{\ln(\eta)} \right]^{\frac{1}{(1-(\mu+\phi))\ln(\eta)-\ln\left(\frac{\sigma_a}{\sigma_m}\right)}} \tag{2.20}$$

and is no longer identical across countries. This open economy TFP effect is used to explain the disparity between (2.19), the density at which a closed economy would industrialize, and the density that a country is observed to industrialize at, (2.20).

V. <u>Empirics</u>

In this section, I consider the two dynamic variables in (2.11), $\left(\frac{\gamma_{a,t}}{\gamma_{m,t}}\right)$ and $\left(\frac{N_t}{L}\right)$, and determine their relative effects in enabling the population density to reach its critical level. To do so I define a new variable, $D_{i,t}$, the distance from industrialization,

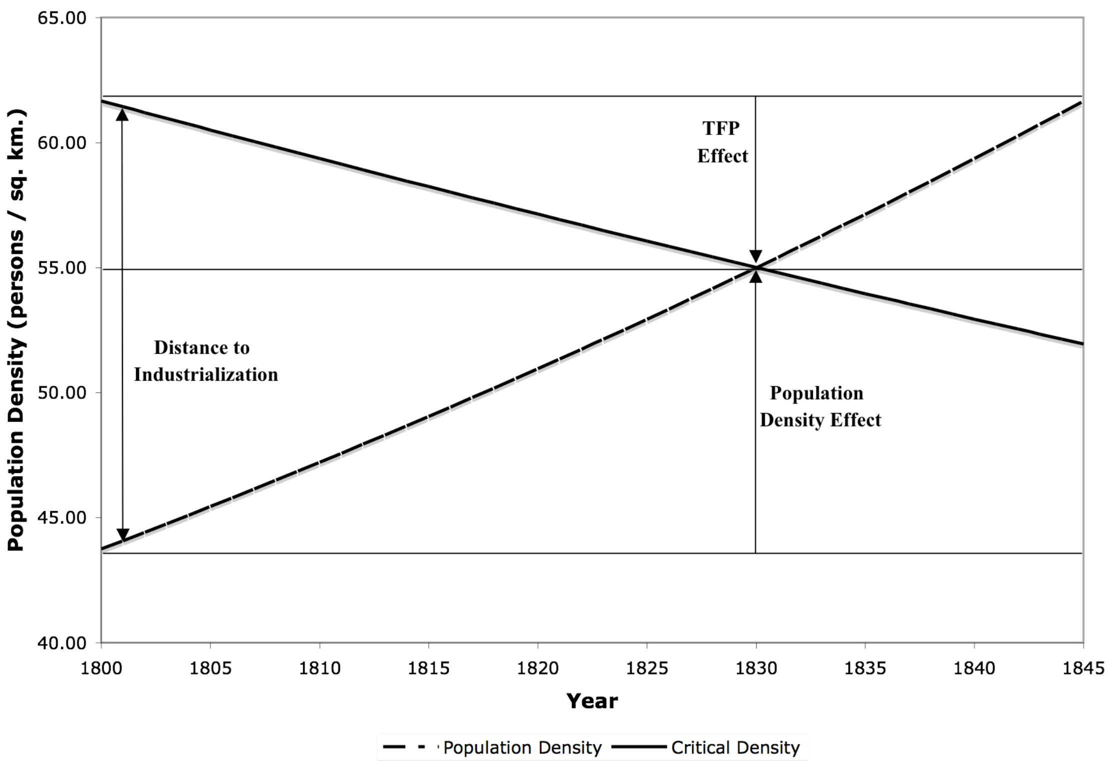
$$D_{i,t} = \left[\left(\frac{\gamma_{a,t}}{\gamma_{m,t}}\right) \left(\frac{\phi}{\theta}\right)^{\theta} \left(\frac{\mu}{1-\theta}\right)^{1-\theta} \left(\frac{1}{k_t}\right)^{\theta-\phi} \right]^{1/1-(\mu+\phi)} - \frac{N_{i,t}}{L} \tag{2.21}$$

where the ratio of agrarian to manufacturing TFP growth, $\left(\frac{\sigma_{a,i}}{\sigma_{m,i}}\right)$, is calibrated so that (2.20) matches the observed density at industrialization and is constant between 1800 and τ_i, the date of industrialization of country i.[14]

[14] This calibration is chosen since it is not possible to directly measure manufacturing TFP prior to the existence of a manufacturing sector. Additionally, historical data is highly inaccurate and there does not exist common accounting across countries for the time periods in question.

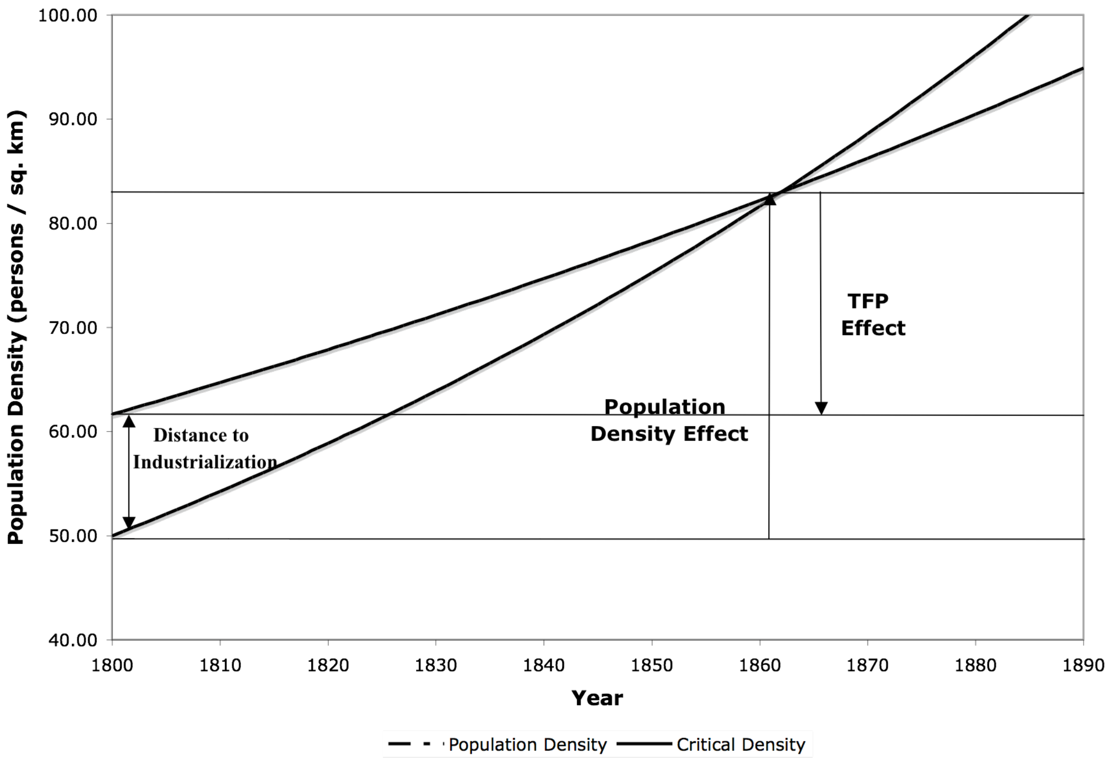
There are two components of change in this distance that occur over time: the TFP effect, caused by $\frac{N_{i,t}}{L}^*$ decreasing due to decreases in $\frac{\gamma_{a,i,t}}{\gamma_{m,i,t}}$, and the population effect, caused by $\frac{N_{i,t}}{L}$ increasing due to population growth. We can decompose movement toward industrialization into these two effects to see the relative effect of each in decreasing $D_{i,t}$.

Figure 2.3: Path to Industrialization: Switzerland



Figures 2.3 and 2.4 depict the TFP effect and population density effect for Switzerland, a country in which TFP aided the path to industrialization, $\frac{\sigma_{a,i}}{\sigma_{m,i}} < 1$, and Germany, a country for which TFP slowed the path to industrialization, $\frac{\sigma_{a,i}}{\sigma_{m,i}} > 1$.

Figure 2.4: Path to Industrialization: Germany



The relative population density effect is defined as the ratio of the increase in population density between 1800 and industrialization to the distance from industrialization in 1800. The model lacks an explanation for why a country would absorb technology in the agrarian sector as opposed to the manufacturing sector (as in Figure 2.4)

The relative population density effect for a cross section of countries is presented in Tables 2.1 and 2.2 for total land and arable land, respectively.

Table 2.1: Population Effect Relative To TFP Effect: Total Land[15]

Country	Approximate Year of Industrialization	Population Growth Rate (1800-Industrialization)	Population Density at 1800 (persons/ sq. km)	Population Density at Industrialization (persons/ sq. km)	Implied Ratio of Agrarian to Industrial TFP Growth Rates (1800-Industrialization)	Relative Population Density Effect (%)
United Kingdom	1800	1.0030	61.67	61.67	-	-
Switzerland	1830	1.0077	43.75	55.00	0.9989	63%
Belgium	1834	1.0056	108.33	131.00	-	-
US	1842	1.0304	3.43	12.07	0.9884	15%
France	1845	1.0044	52.73	64.18	1.0003	128%
Germany	1862	1.0082	50.00	83.00	1.0014	283%
Sweden	1863	1.0070	5.56	8.64	0.9907	6%
Norway	1880	1.0097	2.81	6.09	0.9914	6%
Austria-Hungary	1883	1.0066	36.76	63.47	1.0001	107%
Netherlands	1885	1.0092	66.67	145.00	-	-
Finland	1887	1.0125	2.35	6.94	0.9925	8%
Denmark	1888	1.0087	25.00	53.50	0.9995	78%
Canada	1889	1.0255	0.21	1.96	0.9884	3%
Italy	1896	1.0058	63.33	110.67	-	-
Russia	1903	1.0103	7.55	21.59	0.9969	26%
Spain	1903	1.0048	23.00	37.84	0.9986	38%
Japan	1907	1.0053	75.68	132.97	-	-
Australia	1909	1.0291	0.03	0.59	0.9873	1%
New Zealand	1915	1.0157	0.74	4.44	0.9932	6%
Bulgaria	1920	1.0080	18.18	47.27	0.9993	67%
Portugal	1921	1.0056	30.56	60.22	0.9999	95%
Greece	1923	1.0078	17.31	45.23	0.9992	63%
South Africa	1931	1.0139	1.18	7.24	0.9951	10%
Yugoslavia	1932	1.0081	18.27	53.23	0.9997	81%
Romania	1942	1.0073	22.92	64.75	1.0001	108%
Brazil	1956	1.0211	0.29	7.61	0.9960	12%
Mexico	1957	1.0121	2.75	18.12	0.9977	26%
China	1971	1.0052	34.38	82.90	1.0005	178%
India and Pakistan	1980	1.0082	43.84	191.47	1.0019	828%

[15] Source: Author's own calculations. Approximate years of industrialization were calculated from Bairoch (1982) by matching level of industrialization per capita to UK in 1800. Land area reflects national borders at date of industrialization.

Table 2.2: Population Effect Relative To TFP Effect: Arable Land[16]

Country	Approximate Year of Industrialization	Land Mass (million sq. km)	Population Density 1800 (persons/ sq. km arable land)	Population Density Industrialization (persons/ sq. km arable land)	Implied Ratio of Agrarian to Industrial TFP Growth Rates (1800-Industrialization)	Population Effect (%)
United Kingdom	1800	0.15	265.46	265.46	-	-
Switzerland	1830	0.04	441.47	554.99	-	-
Belgium	1834	0.03	395.09	477.75	-	-
US	1842	1.75	19.04	67.01	0.9902	19%
France	1845	0.55	158.34	192.74	0.9979	32%
Germany	1862	0.36	150.92	250.53	0.9997	87%
Sweden	1863	0.45	81.70	127.12	0.9965	25%
Norway	1880	0.32	104.17	225.69	0.9994	75%
Austria-Hungary	1883	0.17	107.94	186.35	0.9987	50%
Netherlands	1885	0.03	1019.37	2217.13	-	-
Finland	1887	0.34	33.71	99.44	0.9966	28%
Denmark	1888	0.04	44.85	95.98	0.9965	23%
Canada	1889	2.40	4.56	42.85	0.9939	15%
Italy	1896	0.30	239.81	419.03	1.0014	699%
Russia	1903	4.77	105.26	301.16	1.0004	122%
Spain	1903	0.50	84.62	139.22	0.9981	30%
Japan	1907	0.37	623.87	1096.23	-	-
Australia	1909	7.69	0.38	8.62	0.9906	3%
New Zealand	1915	0.27	12.77	76.63	0.9968	25%
Bulgaria	1920	0.11	60.73	157.89	0.9987	47%
Portugal	1921	0.09	176.72	348.31	1.0007	193%
Greece	1923	0.13	84.63	221.18	0.9996	76%
South Africa	1931	1.27	9.76	59.87	0.9966	20%
Yugoslavia	1932	0.26	93.16	271.45	1.0001	103%
Romania	1942	0.24	58.03	163.97	0.9990	51%
Brazil	1956	8.51	4.66	120.72	0.9985	44%
Mexico	1957	2.00	21.72	143.13	0.9988	50%
China	1971	9.60	258.26	622.81	1.0015	5065%
India and Pakistan	1980	4.22	100.16	437.44	1.0008	204%

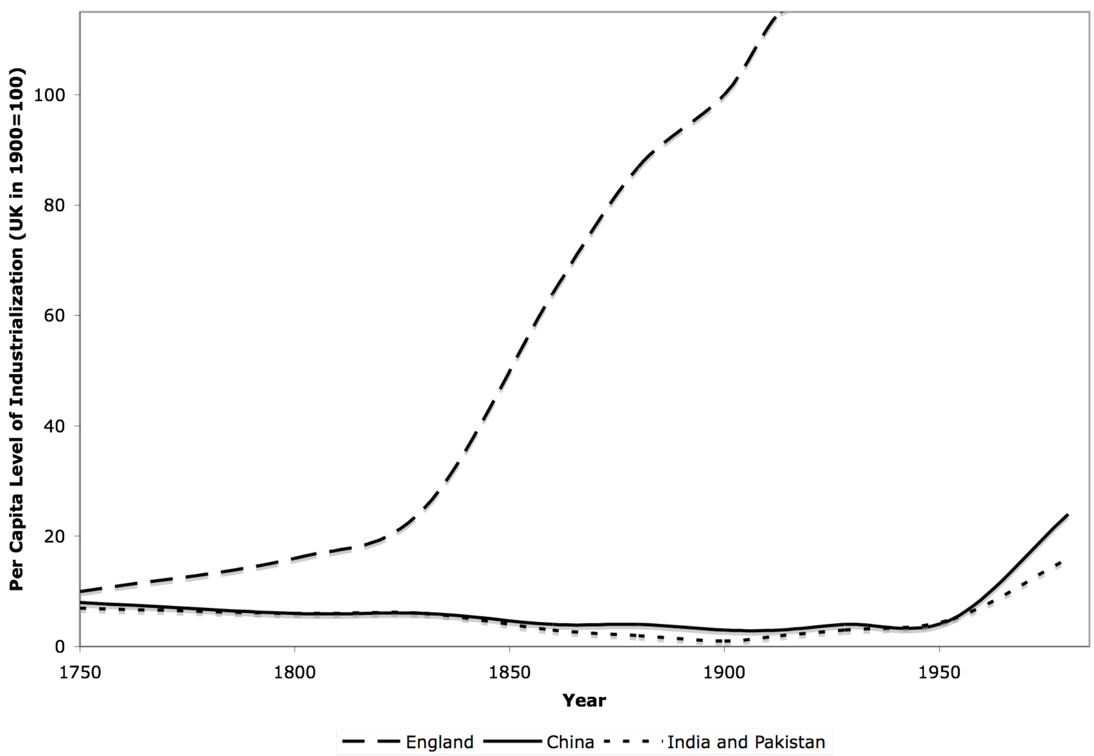
The mean effect across the sample is 93% when total land is considered and increases to 295% when only arable land is taken into account. The means are skewed due to the large population density effects in small number of cases in which the TFP effect is negative. Therefore, the median is a less biased and more accurate representation of the data. The median population density effects are 51% and 49%, respectively.

Figure 2.5 examines China and India, two countries experiencing very large TFP effects. The implication of these effects is they must have absorbed TFP in the agrarian

[16] Agrarian land as a percentage of total land taken from The World Factbook (2002)

rather than the manufacturing sector. This is confirmed in the data, as Figure 2.5 shows that both China and India experienced declines in the per capita level of industrialization. This occurred while output per capita remains constant indicated a decline in manufacturing sector relative to the industrial sector. This is consistent with Tables 2.1 and 2.2, which imply that TFP changes slowed industrialization in these countries.

Figure 2.5: Per Capita Level of Industrialization, China[17]



VI. <u>Conclusion</u>

This paper explores the role of population density in triggering the onset of industrialization. I find that TFP is not as important as others have suggested and that

[17] Per capita level of industrialization taken from Bairoch (1982) and defined as manufacturing output per capita.

population density is capable of accounting for approximately half of the timing of industrialization across countries.

Within the same framework as set for by Hansen and Prescott (2002), I show that the dynamics of population density alone are capable of generating the transition between pre-industrial and industrialized growth paths that are consistent with what is observed in the data.

Furthermore, with additional specifications that are consistent with theory of Lucas (1998) and the calibrations of Parente and Prescott (2004), population density increases are found to account for nearly half of the movement toward industrialization, depending on the measure of land.

Most importantly, this paper provides a way to apply the framework of Hansen and Prescott (2002) to a cross section of countries such that the timing of industrialization across countries is consistent with the data. In the final line of Hansen and Prescott (2002), the authors speculate:

> *The fact that the industrial revolution happened first in England in the early nineteenth century rather than contemporaneously or earlier in China, where the stock of useable knowledge may have actually been higher, is due perhaps to the institutions and policies in place in these two countries.*

Here it is seen that China's lag behind England can be attributed to its lower population density, making it unnecessary to explore the effects of non-measurable quantities such as institutions and policies.

While this paper is focused primarily on the industrial revolution, the ability to express distance to industrialization in terms of two state variables, $\left(\frac{\gamma_{a,t}}{\gamma_{m,t}}\right)$ and $\left(\frac{N_t}{L}\right)$, provides a framework to examine more modern cases of industrialization. Additionally, the relative levels of the two state variables at industrialization may be relevant for explaining the different rates of growth countries experience upon industrialization.

Supplemental Proposition: Pre-Industrial Steady State

Proposition: If there is constant population growth while only the agrarian technology is employed, such that $N_{t+1} = \eta N_t$, and agrarian TFP grows according to $\gamma_{a,t} = \gamma_{a,0}\left(\eta^{1-(\mu+\phi)}\right)^t$ then $\exists$ an equilibrium sequence of land prices such that capital, consumption and output per worker are constant.

Proof: If $\exists$ a series of prices that obey the law of motion for land prices, (2.14), and (2.15), that makes $k_{t+1} = k_t$, then the proposition must be true.

Since all agents in a generation are identical and only the old own land, $l_{t+1} = \frac{L}{N_t}$, while wages are constant such that $\omega = \mu\gamma_{a,0}k^{\phi}\left(\frac{N_0}{L}\right)^{(\mu+\phi)-1}$, it can be seen from (2.15) that a sequence of land prices satisfying $q_{t+1} = \eta q_t$ will make the proposition hold.

The per capita level of capital associated with this steady state is the non-negative level of capital per capita that satisfies (2.15) for $t < \tau$ where

$$q_t = \frac{r_{L,t+1}}{r_{K,t+1} - \eta} \tag{2.22}$$

$$r_{L,t+1} = \left(1 - (\mu + \phi)\right)\gamma_{m,0}\eta^t k^{\phi}\left(\frac{N_0}{L}\right)^{\mu+\phi} \tag{2.23}$$

$$r_{K,t+1} = \phi \gamma_{m,0} k^{\phi-1} \left(\tfrac{N_0}{L}\right)^{(\mu+\phi)-1} \tag{2.24}$$

This level of per capita capital is

$$k_a = \left(\tfrac{N_0}{L}\right)^{\frac{(\mu+\phi)-1}{1-\phi}} Z \tag{2.25}$$

where

$$Z = \left(\frac{\left(1-\mu+\frac{\beta}{1+\beta}(\delta+\eta-1)\right) \pm \sqrt{\left(1-\mu+\frac{\beta}{1+\beta}\mu(\delta+\eta-1)\right)^2 - 4\frac{\beta}{1+\beta}\phi\mu(\delta+\eta-1)}}{2[\delta+\eta-1]} \right)^{1/1-\phi} \tag{2.26}$$

<u>Supplemental Proposition: Conditions on Industrialization</u>

<u>Proposition</u>: For any $\frac{\gamma_{m,t+1}/\gamma_{m,t}}{\gamma_{a,t+1}/\gamma_{a,t}} > \frac{1}{\eta^{1-(\mu+\phi)}}$ or $\frac{\gamma_{m,t+1}/\gamma_{m,t}}{\gamma_{a,t+1}/\gamma_{a,t}} \geq \frac{1}{\eta^{1-(\mu+\phi)}}$, when

$\left(\frac{N_0}{L}\right) > \left[\left(\frac{\gamma_{a,0}}{\gamma_{m,0}}\right) \left(\frac{\phi}{\theta}\right)^{\theta} \left(\frac{\mu}{1-\theta}\right)^{1-\theta} \left(\frac{1}{Z}\right)^{\theta-\phi} \right]^{\frac{\phi-\theta}{(1-\theta)(1-(\mu+\phi))}}$, where $\gamma_{m,t} = \gamma_{m,0}\gamma_m^t$ and $\frac{\gamma_{a,t+1}}{\gamma_{a,t}} = \eta^{1-(\mu+\phi)}$, there exists some $\left(\frac{N}{L}\right)^*$ such that for $\left(\frac{N}{L}\right) > \left(\frac{N}{L}\right)^*$ the industrial sector will be employed.

<u>Proof</u>: From a time at which industrialization has not occurred, in order for (2.11) to become satisfied at some time, it must be that there exists some τ such that $\forall t > \tau$, the left hand side is growing faster than the left hand side. Differentiating (2.11) with respect to time and simplifying, yields the condition that industrialization will eventually occur if

$$\ln(\eta)\left(\tfrac{N_0}{L}\right) > \left(\tfrac{\gamma_{m,t+1}}{\gamma_{m,t}}\right)^{\frac{-t}{1-(\mu+\phi)}} \left(\ln(\eta) - \frac{\ln\left(\gamma_{m,t+1}/\gamma_{m,t}\right)}{1-(\mu+\phi)}\right) \left[\left(\tfrac{\gamma_{a,0}}{\gamma_{m,0}}\right) \left(\tfrac{\phi}{\theta}\right)^{\theta} \left(\tfrac{\mu}{1-\theta}\right)^{1-\theta} \left(\tfrac{1}{k}\right)^{\theta-\phi} \right]^{1/1-(\mu+\phi)} \tag{2.27}$$

Since the right hand side approaches zero for all $\left(\gamma_{m,t+1}/\gamma_{m,t}\right) \geq 1$, the first part of the proposition is proven.

In the case that $\gamma_{m,t+1}/\gamma_{m,t} = 1$, (2.27) reduces to

$$\left(\frac{N_0}{L}\right) > \left[\left(\frac{\gamma_{a,0}}{\gamma_{m,0}}\right)\left(\frac{\phi}{\theta}\right)^{\theta}\left(\frac{\mu}{1-\theta}\right)^{1-\theta}\left(\frac{1}{Z}\right)^{\theta-\phi}\right]^{\frac{\phi-\theta}{(1-\theta)(1-(\mu+\phi))}} \qquad (2.28)$$

proving the second part.

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Chapter Three

Capital Accumulation Prior to Industrialization

ABSTRACT

A sequential competitive equilibrium is presented to solve an infinite horizon two-sector neoclassical growth mode where the two sectors are chosen to represent the agrarian and manufacturing sectors of the economy. In this framework, industrialization is seen to be the relaxing of the non-negativity constrain on the manufacturing sector. It is seen that every country possesses a critical population density upon which it will transition from using solely an agrarian production technology to employing both agrarian and manufacturing technologies. This transition is result of a discrete change in the decision to invest in manufacturing capital. Furthermore, the ability of agents to anticipate industrialization is shown to increase the rate of capital accumulation and hasten the onset of the manufacturing sector.

Keywords: economic development, population density, capital accumulation, industrialization

JEL Codes: N10, N30, O11, O41

In this paper, I explore the role of population density in an infinite horizon version of the unified growth theory of Hansen and Prescott (2002) and show that similar to the overlapped generations case, if a country possesses sufficient population density, the manufacturing technology will always be employed. In addition, anticipation of the implementation of the industrial sector will increase the rate of accumulation of capital and hasten industrialization.

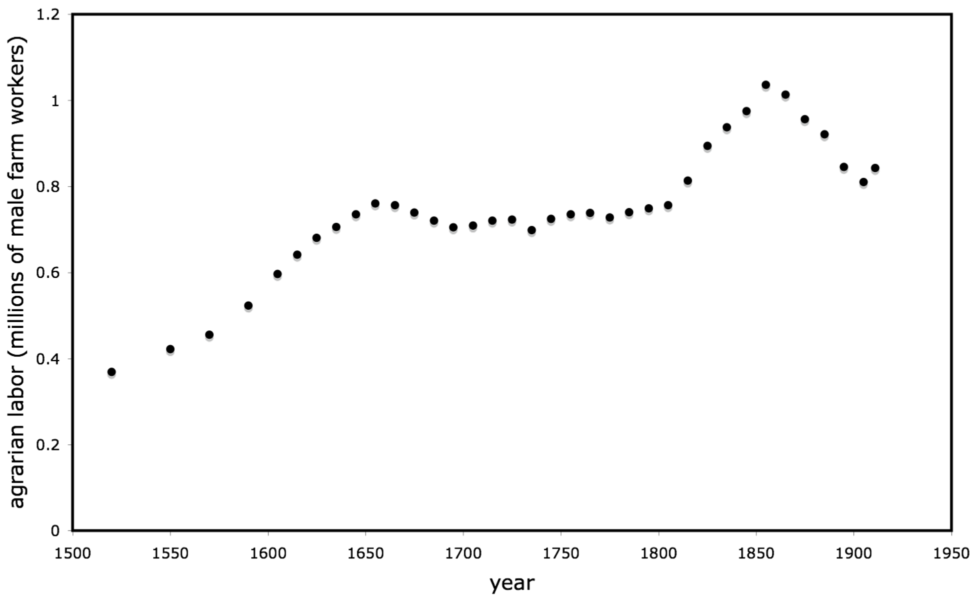
I show that every country possesses a critical population density upon which it will transition from using solely an agrarian production technology to employing both agrarian and manufacturing technologies. This transition is shown to be the result in a change in the investment decision on manufacturing capital. For the purposes of this paper, industrialization is defined as this transition.

The stylized facts typically associated with this literature are two-fold: short run correlations with population and long run time trends. By examining long run correlations with population density, noticeable changes are seen to occur at a critical population density. Figures 3.1, 3.3, and 3.5 present a review of these long run time trends for real wages, land prices, and agrarian labor force in England while Figures 3.2, 3.4, and 3.6 plot the same variables against population density over the same time interval.

While population density is strictly increasing in time, this monotonic transformation provides new insights. For all three variables, a more distinct change in trend is observed at industrialization when population density is the dependent variable.

This lends credibility to the theory that population density is the underlying explanatory variable.

Figure 3.1: Agrarian Labor as a Function of Time[18]



[18] Data is from England 1520-1911 and taken from Clark (2002). Agrarian labor is measured as male farm workers (in millions). Population is measured in millions of persons. Wage (of farm workers) and rental rate of (farm) land are both normalized to 100 in 1865.

Figure 3.2: Agrarian Labor as a Function of Population Density

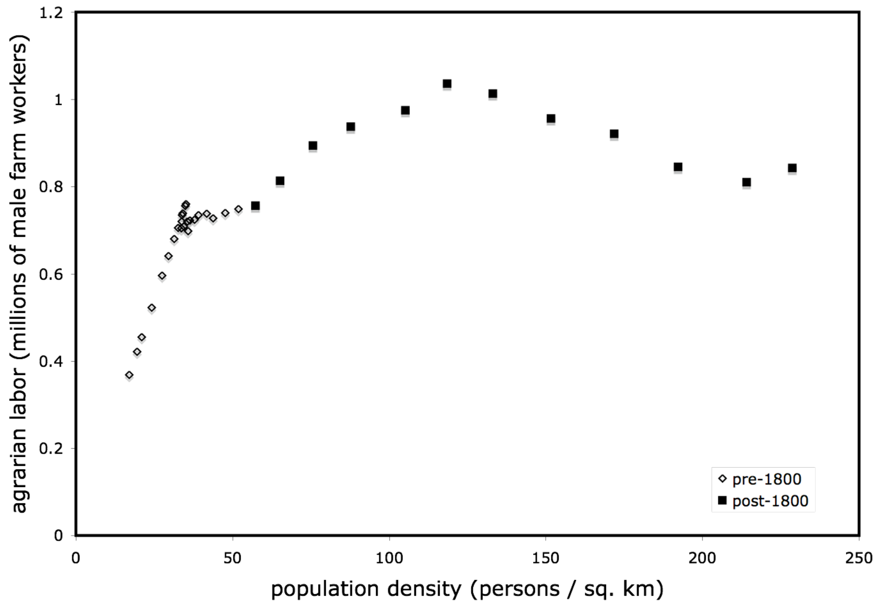


Figure 3.3: Rental Rate of Land as a Function of Time

Figure 3.4: Rental Rate of Land as a Function of Population Density

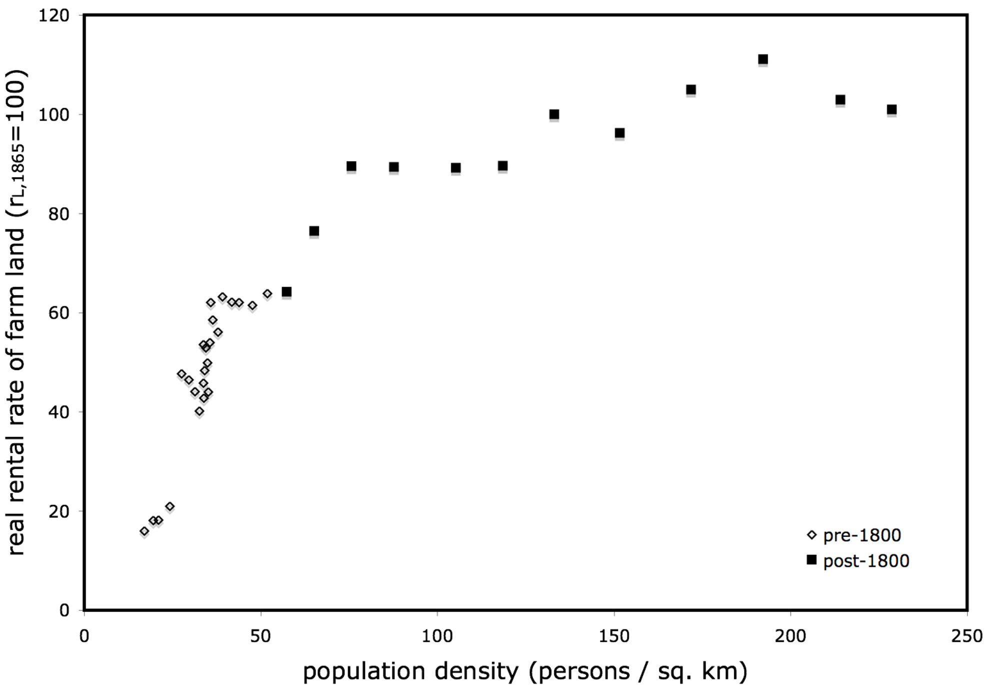


Figure 3.5: Real Wage as a Function of Time

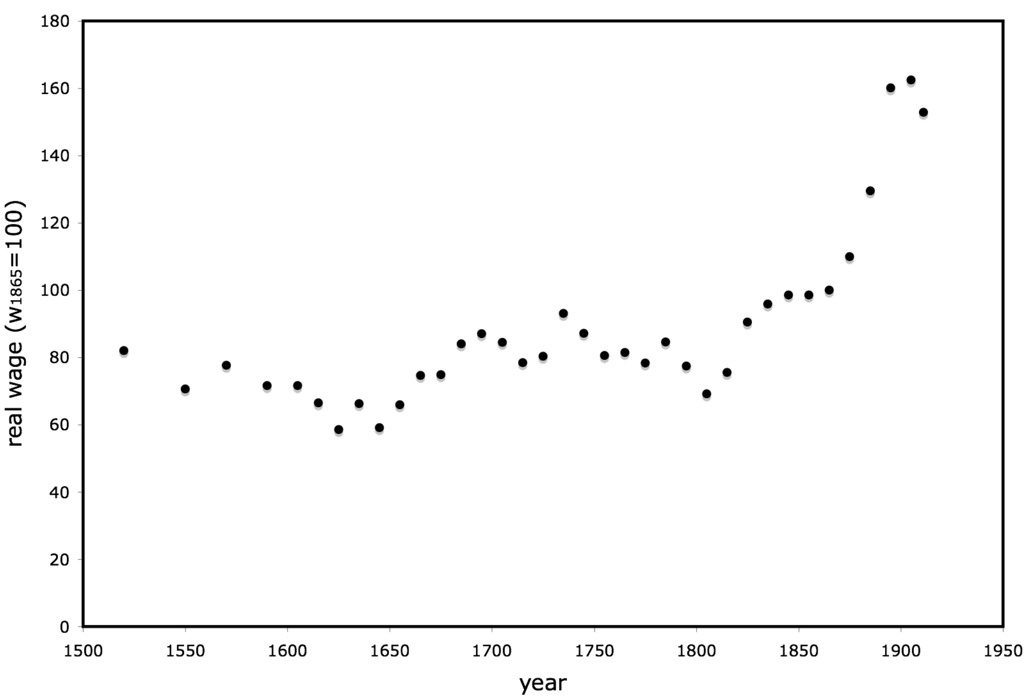
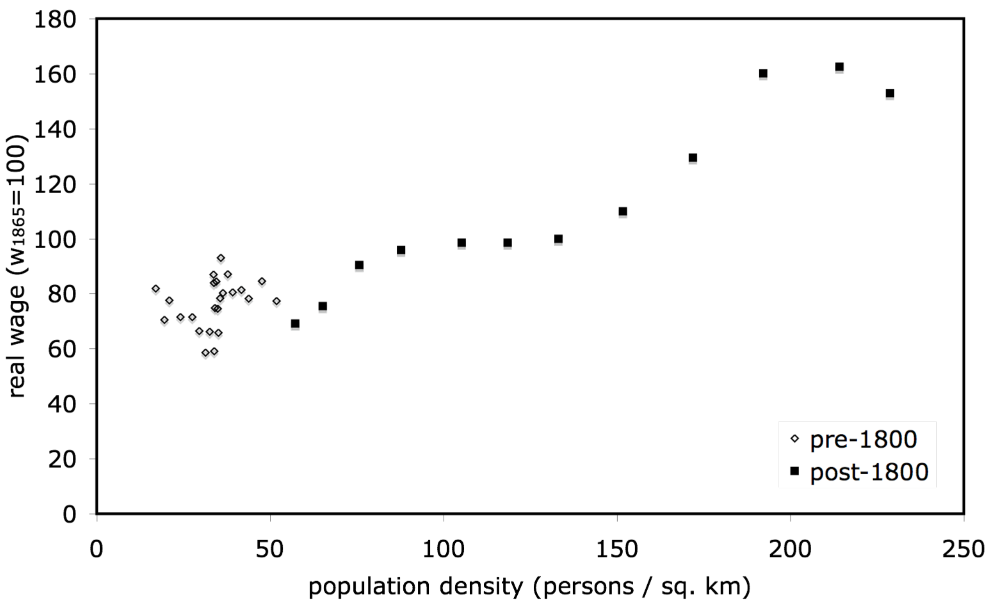


Figure 3.6: Real Wage as a Function of Population Density



The next section describes the environment. Section III presents a sequential equilibrium for a simplified version of the model and Section IV presents the full sequential equilibrium. In Section V, a numerical solution algorithm is presented and the results shown. Section VI concludes.

II. Environment

This closed model economy is an infinitely lived agent variation of the overlapping generation model described by Hansen and Prescott (2002) and the previous chapters.

There are three resources in the economy: land, labor, and capital. There are two types of firms: agrarian firms that have access to a Malthusian production technology, $Y_{a,t} = \gamma_{a,t} K_{a,t}^{\phi} N_{a,t}^{\mu} L_{a,t}^{1-\mu-\phi}$, that uses capital, labor, and land, and industrial firms that have access to a Solow production technology, $Y_{m,t} = \gamma_{m,t} K_{m,t}^{\theta} N_{m,t}^{1-\theta}$, that utilizes only capital and labor. Give prices $r_{L,t}$, $r_{Ka,t}$ and $\omega_{a,t}$, agrarian firms demand L_t units of land, $K_{a,t}$ units of capital, and $N_{a,t}$ units of labor, respectively, at the beginning of each period.

Similarly, given prices $r_{Km,t}$ and $\omega_{m,t}$, industrial firms demand $K_{m,t}$ units of capital and $N_{m,t}$ units of labor. Both production technologies produce an identical aggregate good and all firms face perfectly competitive markets for labor, land, and capital, where capital depreciates by δ each period. There are no fixed costs and no savings technology is available to firms. Therefore, each sector can be reduced to a single representative firm that solves a static maximization problem and operates only in periods where profits are nonnegative. TFP in sector $i = a,m$ evolves according to $\gamma_{i,t} = \gamma_{i,0} \cdot \gamma^t$ where $\gamma = \eta^{1-(\mu+\phi)}$.

There is a continuum of ex-ante identical, infinitely lived agents with preferences $\sum_{t=0}^{\infty} \beta^t \ln(c_t)$. New agents are born each period such that $N_{t+1} = \eta N_t$. Agents enter each period with a stock of agrarian capital, $k_{a,t}$, industrial capital, $k_{m,t}$, land, l_t, and a unit supply of labor rent $l_t \in X$ units of land, $k_{a,t}$ units of capital, and $n_{a,t} \in [0,1]$ units of labor to agrarian firms; rent capital $k_{m,t}$ and $n_{m,t} \in [0,1]$ units of labor to industrial firms; consume c_t; and choose $\{i_{L,t}, i_{Ka,t}, i_{Km,t}\} \in \mathbb{R}^3$ to maximize lifetime utility, where the initial values, $\{q_0 l_0, K_{a,0}, K_{m,0}, N_0\} \in X \subset \mathbb{R}^4_+$, are given and where X is assumed to be closed and convex.

III. Simple Example

In this simple version of the model, there is no capital, $\phi = \theta = 0$, and the representative agrarian firm solves the sequence of static problems

$$\max_{N_{a,t},L_t} \gamma_{a,t} N_{a,t}^{\mu} L_t^{1-\mu} - \omega_{a,t} N_{a,t} - r_{L,t} L_t \tag{3.1}$$

while the representative industrial firm solves the sequence of static problems

$$\max_{N_{m,t}} \gamma_{m,t} N_{m,t} - \omega_{m,t} N_{m,t} \tag{3.2}$$

Due to constant returns to scale, there are no profits and the optimization problems yield factor prices of

$$\omega_{a,t} = \mu \gamma_{a,t} \left(\tfrac{N_{a,t}}{L_t}\right)^{\mu-1} \tag{3.3}$$

$$\omega_{m,t} = \gamma_{m,t} \tag{3.4}$$

$$r_{L,t} = (1-\mu) \gamma_{a,t} \left(\tfrac{N_{a,t}}{L_t}\right)^{\mu} \tag{3.5}$$

Given $q_0 l_0$ and $\{N\}_{t=0}^{\infty}$, agents take prices $\{q_t, \omega_{a,t}, \omega_{m,t}, r_{L,t}\}_{t=0}^{\infty}$ as given and solve

$$\begin{aligned}
&\max_{n_{a,t},n_{m,t},c_t,l_{t+1}} \sum_{t=0}^{\infty} \beta^t \ln(c_t) \\
&s.t. \\
&c_t + q_t l_{t+1} = \omega_{a,t} n_{a,t} + \omega_{m,t} n_{m,t} + (r_{L,t} + q_t) l_t \\
&0 \le n_{a,t}, n_{m,t} \le 1 \\
&n_{a,t} + n_{m,t} = 1 \\
&given: q_0 l_0, \{N_t\}_{t=0}^{\infty}
\end{aligned} \tag{3.6}$$

A *sequential competitive equilibrium* is a price sequence $\left\{q_t,\omega_{a,t},\omega_{m,t},r_{L,t}\right\}_{t=0}^{\infty}$ and an allocation $\left\{c_t,n_{a,t},n_{m,t},l_t\right\}_{t=0}^{\infty}$ such that households, (3.6), and firms, (3.1) and (3.2), optimize and markets clear:

$$C_t=\gamma_{a,t}N_{a,t}^{\mu}L_t^{1-\mu}+\gamma_{m,t}N_{m,t} \tag{3.7}$$

$$N_t l_t=\overline{L} \tag{3.8}$$

$$N_t n_{a,t}=N_{a,t} \tag{3.9}$$

$$N_t n_{m,t}=N_{m,t} \tag{3.10}$$

The market clearing conditions also represent a rational expectations requirement for individuals.

The relevant state variables for an agent in period t are his current stock of land, l_t, the aggregate stock of land, L_t, and the total number of agents, N_t. When capital is reintroduced into the problem, each agent's stock of agrarian capital, $k_{a,t}$, and manufacturing capital, $k_{m,t}$; and the aggregate agrarian capital, $K_{a,t}$, and manufacturing capital, $K_{m,t}$ will also be relevant.

Taking first order conditions of (3.6) gives:

$$\tfrac{q_t}{c_t}=\beta\tfrac{q_{t+1}+r_{L,t+1}}{c_{t+1}}+\lambda_{L,t} \tag{3.11}$$

$$\tfrac{\omega_{a,t}-\omega_{m,t}}{c_t}+\lambda_{na,t}-\lambda_{nm,t}=0 \tag{3.12}$$

where $\lambda_{L,t}$ is the multiplier on the constraint $l_{t+1}\geq 0$, $\lambda_{na,t}$ is the multiplier on the constraint $n_{a,t}\geq 0$, and $\lambda_{nm,t}$ is the multiplier on the constraint $1-n_{a,t}\geq 0$.

From market clearing, it is not possible for $l_{t+1} = 0$, and $l_{t+1} = \frac{L_{t+1}}{N_{t+1}}$.

Lemma 1: If $L_t > 0$, then $n_{a,t} > 0$.

Proof: Suppose that $n_{a,t} = 0$. Then $\lambda_{na,t} > 0$ and as a result, $\omega_{a,t} - \omega_{m,t} < 0$ and $N_{a.t} = 0$. Substituting in the wages from the firm problem, this requires that $\lim_{N_a \to 0} \gamma_{m,t} > \mu\gamma_{a,t}\left(\frac{N_a}{L_t}\right)^{\mu-1}$. However, for all $L_t > 0$, the right hand side is infinite, while the left hand side is bounded. Thus, $\forall\ L_t > 0$ it must be that $\lambda_{na,t} = 0$ and $n_{a,t}, N_{a,t} > 0$.

q.e.d.

Lemma 2: There exists some $\left(\frac{N_t}{L_t}\right)$ such that when $\gamma_a = \gamma_m$ and $\left(\frac{N_t}{L_t}\right) > \left[\mu\frac{\gamma_{a,0}}{\gamma_{m,0}}\right]^{\frac{1}{1-\mu}}$, $n_{a,t} < 1$.

Proof: Suppose that $n_{a,t} = 1$, then $\lambda_{nm,t} > 0$, $\omega_{a,t} - \omega_{m,t} > 0$, and $N_{a.t} = N_t$. Substituting in the wages from the firm problem, this requires

$$\mu\gamma_{a,0}\left(\frac{N_t}{L_t}\right)^{\mu-1} > \gamma_{m,0} \tag{3.13}$$

Since the left hand side is decreasing in $\left(\frac{N_t}{L_t}\right)$, there must exist a minimum $\left(\frac{N_t}{L_t}\right)$ at which (3.13) can no longer hold. *q.e.d.*

This gives the condition that the constraint on manufacturing capital is not binding when

$$\left(\frac{N_t}{L_t}\right) > \left[\mu\frac{\gamma_{a,t}}{\gamma_{m,t}}\right]^{\frac{1}{1-\mu}} \tag{3.14}$$

Combing this result with Lemma 1, (3.14) is the condition for which nether constraint is binding and both technologies are in use.

For the case when $\gamma_{a,t} = \gamma_{m,t}$, (3.14) reduces to $\frac{N_t}{L_t} > \mu^{\frac{1}{1-\mu}}$, which will be seen to be the same condition derived from the overlapping generations model of chapter two.

Given that both technologies are in use, $\lambda_{na,t} = 0$ and $\lambda_{nm,t} = 0$, (3.14) is satisfied, and it must be that $\omega_{a,t} - \omega_{m,t} = 0$. We therefore have

$$N_{a,t} = L\left(\mu \frac{\gamma_{a,t}}{\gamma_{m,t}}\right)^{\frac{1}{1-\mu}} \tag{3.15}$$

Taking (3.14) and (3.15) as given, households are indifference about $n_{a,t} \in [0,1]$. Thus there exists an equilibrium where there is consistency of aggregate and individual decisions, (3.9), and

$$N_{a,t} = \begin{cases} N_t & \text{if } \frac{N_t}{L_t} \le \left(\mu \frac{\gamma_{a,t}}{\gamma_{m,t}}\right)^{\frac{1}{1-\mu}} \\ L_t\left(\mu \frac{\gamma_{a,t}}{\gamma_{m,t}}\right)^{\frac{1}{1-\mu}} & \text{if } \frac{N_t}{L_t} > \left(\mu \frac{\gamma_{a,t}}{\gamma_{m,t}}\right)^{\frac{1}{1-\mu}} \end{cases} \tag{3.16}$$

IV. <u>Sequential Competitive Equilibrium</u>

With the return of capital, the representative agrarian firm solves the sequence of one period problems

$$\max_{K_{a,t}N_{a,t},L_t} \gamma_{a,t} K_{a,t}^{\phi} N_{a,t}^{\mu} L_t^{1-(\mu+\phi)} - r_{ka,t}K_{a,t} - \omega_{a,t}N_{a,t} - r_{L,t}L_t \tag{3.17}$$

while the representative industrial firm solves the sequence of one period problems

$$\max_{K_{m,t},N_{m,t}} \gamma_{m,t} K_{m,t}^{\theta} N_{m,t}^{1-\theta} - r_{Km,t}K_{m,t} - \omega_{m,t}N_{m,t} \tag{3.18}$$

Due to constant returns to scale, there are no profits and factors will earn a rent equal to their marginal product.

Given $\{q_0 l_0, k_{a,0}, k_{m,0}\}$ and $\{N_t\}_{t=0}^{\infty}$, households take prices $\{q_t, \omega_{a,t}, \omega_{m,t}, r_{L,t}, r_{Ka,t}, r_{Km,t}\}_{t=0}^{\infty}$ as given and solve

$$\begin{aligned}
&\max_{n_{a,t}, n_{m,t}, c_t, l_{t+1}, k_{a,t+1}, k_{m,t+1}} \sum_{t=0}^{\infty} \beta^t \ln(c_t) \\
&s.t. \\
&c_t + q_t l_{t+1} + k_{a,t+1} + k_{m,t+1} = \omega_{a,t} n_{a,t} + \omega_{m,t} n_{m,t} + \\
&(r_{Ka,t} + 1 - \delta) k_{a,t} + (r_{Km,t} + 1 - \delta) k_{m,t} + (r_{L,t} + q_t) l_t \\
&0 \le n_{a,t}, n_{m,t} \le 1 \\
&n_{a,t} + n_{m,t} = 1
\end{aligned} \tag{3.19}$$

A *sequential competitive equilibrium* is a price sequence $\{q_t, \omega_{a,t}, \omega_{m,t}, r_{L,t}, r_{Ka,t}, r_{Km,t}\}_{t=0}^{\infty}$ and an allocation $\{c_t, n_{a,t}, n_{m,t}, k_{a,t}, k_{m,t}, l_t\}_{t=0}^{\infty}$ such that households, (3.19), and firms, (3.17) and (3.18), optimize and markets clear; (3.8), (3.9), (3.10),

$$C_t + K_{a,t+1} + K_{m,t+1} = \gamma_{a,t} K_{a,t}^{\phi} N_{a,t}^{\mu} L_t^{1-(\mu+\phi)} + \gamma_{m,t} K_{m,t}^{\theta} N_{m,t}^{1-\theta} + (1-\delta)(K_{a,t} + K_{m,t}) \tag{3.20}$$

$$N_t k_{a,t} = K_{a,t} \tag{3.21}$$

$$N_t k_{m,t} = K_{m,t} \tag{3.22}$$

Taking the first order conditions of (3.19) and substituting in some of the constraints gives:

$$\tfrac{q_t}{c_t} = \beta \tfrac{q_{t+1} + r_{L,t+1}}{c_{t+1}} + \lambda_{L,t} \tag{3.23}$$

$$\tfrac{\omega_{a,t} - \omega_{m,t}}{c_t} + \lambda_{na,t} - \lambda_{nm,t} = 0 \tag{3.24}$$

$$\tfrac{1}{c_t} = \beta \tfrac{r_{Ka,t+1}+1-\delta}{c_{t+1}} + \lambda_{ka,t} \tag{3.25}$$

$$\tfrac{1}{c_t} = \beta \tfrac{r_{Km,t+1}+1-\delta}{c_{t+1}} + \lambda_{km,t} \tag{3.26}$$

where $\lambda_{ka,t}$ is the multiplier on the constraint; $k_{a,t+1} \geq 0$, $\lambda_{km,t}$ is the multiplier on the constraint $k_{m,t+1} \geq 0$; and $\lambda_{L,t}$, $\lambda_{na,t}$, and $\lambda_{nm,t}$ follow from the simple example.

Lemma 3: Given that $\frac{c_{t+1}}{c_t}$ is bounded and $L_t > 0$ for all t, then $n_{a,t} > 0$ and $k_{a,t+1} > 0$.

Proof: Suppose that $n_{a,t} = 0$. Then $\lambda_{na,t} > 0$ and $\omega_{a,t} - \omega_{m,t} < 0$ and $n_{a,t} = N_{a.t} = 0$. Substituting in the price rules, this condition requires that $\lim_{N_a \to 0} (1-\theta) \cdot \gamma_{m,t} \cdot K_{m,t}^{\theta} \cdot (N_t - N_a)^{-\theta} - \mu \cdot \gamma_{a,t} \cdot K_{a,t}^{\phi} \cdot N_a{}^{\mu-1} \cdot L_t^{1-(\mu+\phi)} > 0$. However, for all $L_t > 0$ or $K_{a,t} > 0$, the second term is infinite, while the first term is bounded. Therefore, it must be that $\lambda_{na,t} = 0$ and $n_{a,t} > 0$.

Similarly, suppose that for some t, $k_{a,t+1} = 0$. Then $\lambda_{ka,t} > 0$, $\frac{1}{c_t} > \beta \frac{r_{Ka,t+1}+1-\delta}{c_{t+1}}$, and $K_{a,t+1} = 0$. This gives the result that $\lim_{K_a \to 0} \phi \cdot \gamma_{a,t+1} \cdot K_a^{\phi-1} \cdot N_{a,t+1}^{\mu} \cdot L_{t+1}^{1-(\mu+\phi)} + 1 - \delta < \frac{c_{t+1}}{\beta c_t}$. So long as $L_{t+1} > 0$, the left hand side is infinite while the right hand side is bounded so it is the case that investment in agrarian capital is always positive and $\lambda_{ka,t} = 0$ and $k_{a,t+1} > 0$.

q.e.d.

Therefore $\frac{1}{c_t} = \beta \frac{r_{Ka,t+1}+1-\delta}{c_{t+1}}$ and $K_{a,t+1}$ is the unique K_a that solves

$$\phi \cdot \gamma_{a,t+1} \cdot K_{a,t+1}^{\phi-1} \cdot N_{a,t+1}^{\mu} \cdot L_{t+1}^{1-(\mu+\phi)} + 1 - \delta = \tfrac{c_{t+1}}{\beta c_t} \tag{3.27}$$

and when agents have rational expectations of $K_{a,t+1}$ and $K_{m,t+1}$, $k_{a,t+1} = \frac{K_{a,t+1}}{N_{t+1}}$.

Lemma 4: If $K_{m,t} > 0$, then $n_{m,t} > 0$.

Proof: Suppose that $n_{m,t} = 0$ and $K_{m,t} > 0$. Then $\lambda_{na,t} = 0$, which implies $\omega_{a,t} - \omega_{m,t} > 0$ and $N_{a,t} = N_t$. Substituting in the price rules, this requires $\lim_{N_a \to N_t} (1-\theta)\cdot\gamma_{m,t}\cdot K_{m,t}^{\theta}\cdot N_a^{-\theta} - \mu\cdot\gamma_{a,t}\cdot K_{a,t}^{\phi}\cdot N_a^{\mu-1}\cdot L_t^{1-(\mu+\phi)} < 0$. As in the agrarian sector, if $K_{m,t} > 0$, the first term is infinite while the second term is bounded and it is the case that $n_{m,t} > 0$ whenever $K_{m,t} > 0$. *q.e.d.*

When neither labor constraint is binding, both technologies are in use, and wages are equalized across sectors. We then have $N_{a,t}$ is the unique N_a that solves:

$$(1-\theta)\cdot\gamma_{m,t}\cdot K_{m,t}^{\theta}\cdot\left(N_t - N_a\right)^{-\theta} = \mu\cdot\gamma_{a,t}\cdot K_{a,t}^{\phi}\cdot N_a^{\mu-1}\cdot L_t^{1-(\mu+\phi)} \tag{3.28}$$

Using these results, we can obtain an aggregate decision rule for labor allocation.

$$N_{a,t} = \begin{cases} 0 & if \quad K_{a,t} = 0 \\ 1 & if \quad K_{m,t} = 0 \\ N_a & otherwise \end{cases} \tag{3.29}$$

where N_a solves (3.28) and $n_{a,t}$ follows from (3.9).

Since non-zero labor will always be allocated to a sector with non-zero capital, the decision rule on $n_{a,t}$ is not the condition that determines industrialization. Rather it is the capital investment decisions that determine whether production occurs in a given sector in the future periods.

Lemma 5: Given that $\frac{c_{t+1}}{c_t}$ is bounded, there exists a critical density that varies in time and capital stock, $d(t,K)$, such that if $N_t/L_t > d(t,K)$, then $k_{m,t+1} > 0$, else $k_{m,t+1} = 0$.

Proof: Suppose $K_{m,t+1} = 0$. Then $\frac{1}{c_t} > \beta \frac{r_{Km,t}+1-\delta}{c_{t+1}}$. It therefore must be the case that $\lim_{K_{m,t} \to 0^+} \frac{1}{c_t} - \beta \frac{r_{Km,t}+1-\delta}{c_{t+1}} > 0$. Since from Lemma 3, we know $K_{a,t+1} > 0$; in the limit $K_{m,t+1} > 0$ and the return capital must be equal across sectors

$$\theta \cdot \gamma_{m,t} \cdot K_{m,t}^{\theta-1} \cdot \left(N_t - N_a\right)^{1-\theta} = \phi \cdot \gamma_{a,t} \cdot K_{a,t}^{\phi-1} \cdot N_a^{\mu} \cdot L_t^{1-(\mu+\phi)} \tag{3.30}$$

as must the return to labor, (3.28).

Dividing (3.30) by (3.28) and rearranging, it is possible to obtain agrarian and manufacturing labor as functions of the aggregate capital, agrarian capital, and aggregate labor.

$$N_{a,t} = \frac{N_t K_{a,t}}{K_t\left(\frac{\phi}{\mu}\right)\left(\frac{1-\theta}{\theta}\right) + \left(1 - \left(\frac{\phi}{\mu}\right)\left(\frac{1-\theta}{\theta}\right)\right)K_{a,t}} \tag{3.31}$$

$$N_{m,t} = \frac{\left(\frac{\phi}{\mu}\right)\left(\frac{1-\theta}{\theta}\right)N_t\left(K_t - K_{a,t}\right)}{K_t\left(\frac{\phi}{\mu}\right)\left(\frac{1-\theta}{\theta}\right) + \left(1 - \left(\frac{\phi}{\mu}\right)\left(\frac{1-\theta}{\theta}\right)\right)K_{a,t}} \tag{3.32}$$

Substituting in for $r_{Km,t}$ and using (3.31) and (3.32), it can be seen that $\lambda_{m,t} > 0$ if and only if

$$\frac{N_t}{L} < \left[\left(\frac{1}{\theta\gamma_{m,t+1}}\right)\left(\frac{c_{t+1}}{\beta c_t} - (1-\delta)\right)\right]^{1/1-\theta} \left(\frac{K_{t+1}}{\eta L}\right)\left(\frac{\mu}{\phi}\right)\left(\frac{\theta}{1-\theta}\right) \tag{3.33}$$

q.e.d.

This is the corresponding condition to the critical population density from chapter two.

For cases when the constraint is not binding, $\frac{N}{L} \geq \frac{N}{L}^{*}$, $\lambda_{km,t} = 0$, and industrial capital investment, $K_{m,t+1}$, is the unique K_m that solves

$$\theta \cdot \gamma_{m,t+1} \cdot K_{m,t+1}^{\theta-1} \cdot N_{m,t+1}^{1-\theta} + 1 - \delta = \frac{c_{t+1}}{\beta c_t} \tag{3.34}$$

Similarly, $K_{m,t+1}$ can also be determined from (3.27) and (3.30).

V. <u>Unbalanced Growth Path</u>

A balanced growth path does not exist for this economy. As seen in Landon-Lane and Robertson (2005), Restuccia (2004), and Caselli and Coleman (2001), a balanced growth path only exists in the case that TFP growth in the agrarian sector exceeds that of the industrial sector. Instead, the economy will approach a balanced growth path only in the limit as population density gets sufficiently large.

When N_t/L_t is large enough so that both technologies are employed, (3.28) and (3.30) both hold, $N_a(N_t, K_t)$ and $N_m(N_t, K_t)$ follow from (3.31) and (3.32), and it can be shown that upon industrialization, in the special case where the labor share of output is identical across sectors, the aggregate supply of agrarian capital evolves with the ratio of TFP across sectors

$$K_{a,t>\tau} = \left(\left(\frac{\gamma_{a,t}}{\gamma_{m,t}} \right) \left(\frac{\phi}{\theta} \right)^{\theta} \left(\frac{\mu}{1-\theta} \right)^{1-\theta} \right)^{\frac{1}{1-(\mu+\phi)}} L \tag{3.35}$$

Furthermore, since (3.27) always holds, the sequence for the aggregate supply of capital, $\{K_t\}_{t=0}^{\infty}$, where $K_t = K_{a,t} + K_{m,t}$, obeys

$$\frac{\gamma_{a,t+1}N_{a,t+1}^{\mu}K_{a,t+1}^{\phi}L^{1-(\mu+\phi)}+\gamma_{m,t+1}\left(\eta^{t+1}N_0-N_{a,t+1}\right)^{1-\theta}\left(K_{t+1}-K_{a,t+1}\right)^{\theta}+\left(1-\delta\right)K_{t+1}-K_{t+2}}{\gamma_{a,t+1}N_{a,t}^{\mu}K_{a,t}^{\phi}L^{1-(\mu+\phi)}+\gamma_{m,t+1}\left(\eta^{t}N_0-N_{a,t}\right)^{1-\theta}\left(K_{t}-K_{a,t}\right)^{\theta}+\left(1-\delta\right)K_{t}-K_{t+1}}$$
$$=\eta\beta\left(\gamma_{a,t+1}N_{a,t+1}^{\mu}K_{a,t+1}^{\phi}L^{1-(\mu+\phi)}+1-\delta\right) \tag{3.36}$$

for all t.

The limiting condition

$$\tfrac{K_{T+1}}{K_T}=\gamma_m^{1-(\mu+\phi)/1-\theta} \tag{3.37}$$

follows from the assumption that the model approaches a purely manufacturing economy in the limit. Given $\{K_0,N_0\},\{N_t,\gamma_{a,t},\gamma_{m,t}\}_{t=0}^{T+1}$, (3.37) in conjunction with (3.36) ($t=1:T$) and (3.28) and (3.30) when $K_t \geq K_{a,t>\tau}$ or $K_{a,t}=K_t$ and $N_{a,t}=N_t$ for $K_t < K_{a,t>\tau}$ ($t=0:T+1$) provides the $3T+5$ equations needed to find the $3T+5$ unknowns, $\{K_t\}_{t=1}^{T+1},\{K_{a,t},N_{a,t}\}_{t=0}^{T+1}$. With the exception of $\gamma_{i,t}$, which was specified previously, the parameters follow from Parente and Prescott (2004): $\theta=.4$, $\phi=.1$, $\mu=.6$, $\beta=.97$, $\delta=.06$.[19]

Figures 3.7 and 3.8 depict the growth paths of capital and output respectively, broken down by sectors. From this diagram it can be seen that that in the limit, the multi-sector economy does approach that of a purely manufacturing economy, justifying the limiting condition of the solution algorithm. It is also interesting to note that in the limit,

[19] Parente and Prescott (2004) set $\gamma_s=\gamma_a$ prior to industrialization and $\gamma_s>\gamma_a$ following industrialization. Since the data, from Bairoch (1981), and theory, Lucas (2002), imply that TFP should increase more rapidly in both sectors following industrialization, equality is assumed.

the capital to labor ratio in the manufacturing sector approaches a level four times that of the agrarian sector.

Figure 3.7: Growth Path of Capital

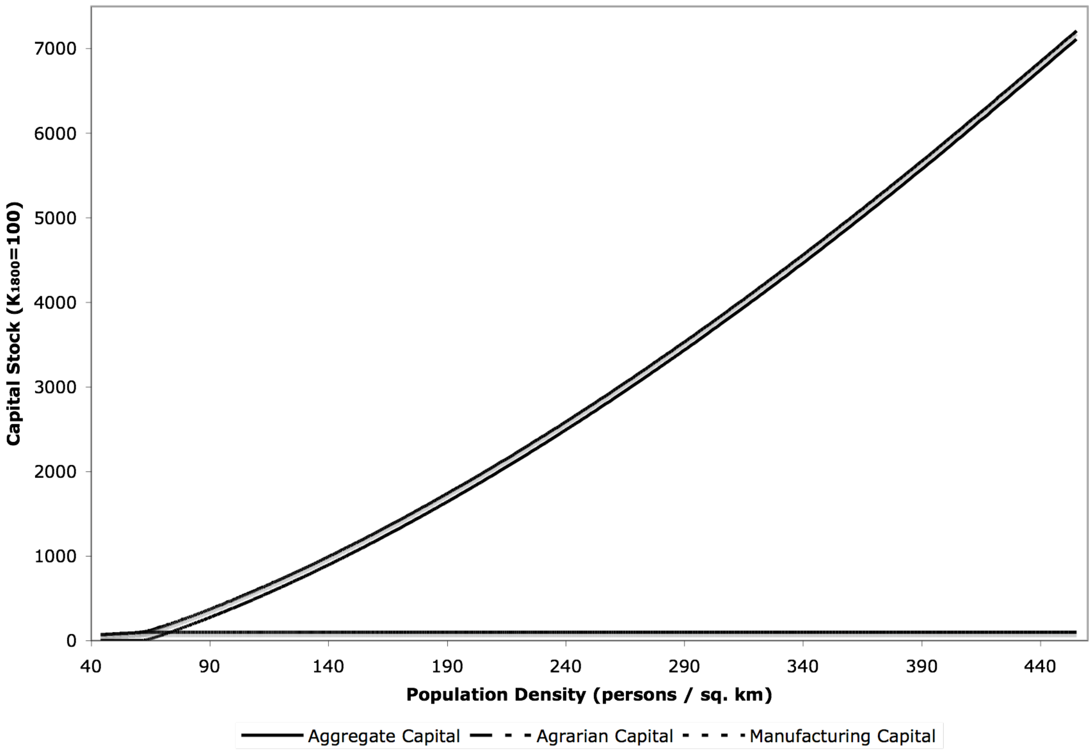
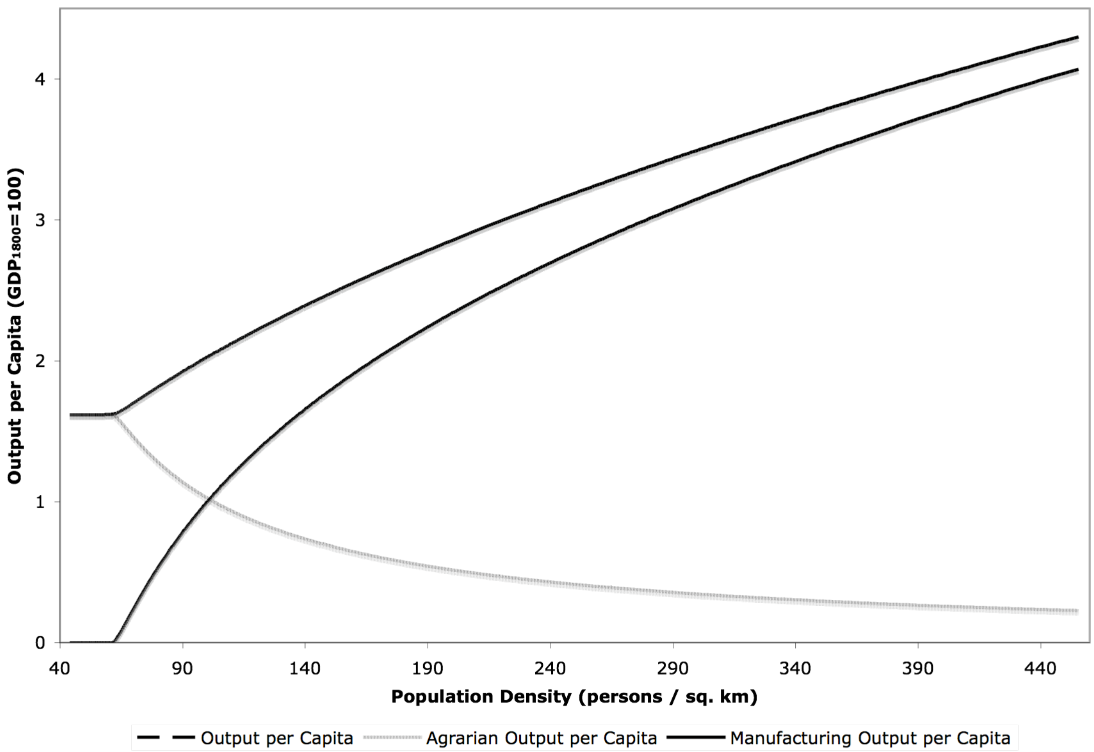


Figure 3.8: Output per Capita



In contrast to chapter two, capital per capita is not constant prior to industrialization. Instead, agents increase the rate of capital accumulation in anticipation of the manufacturing technology, hastening the onset of industrialization. Figure 3.9 depicts this by contrasting the capital path in this model to one in which agents lack foresight, or in which capital depreciates fully as in the prior chapters.

Figure 3.9: Capital Accumulation, Effect of Foresight

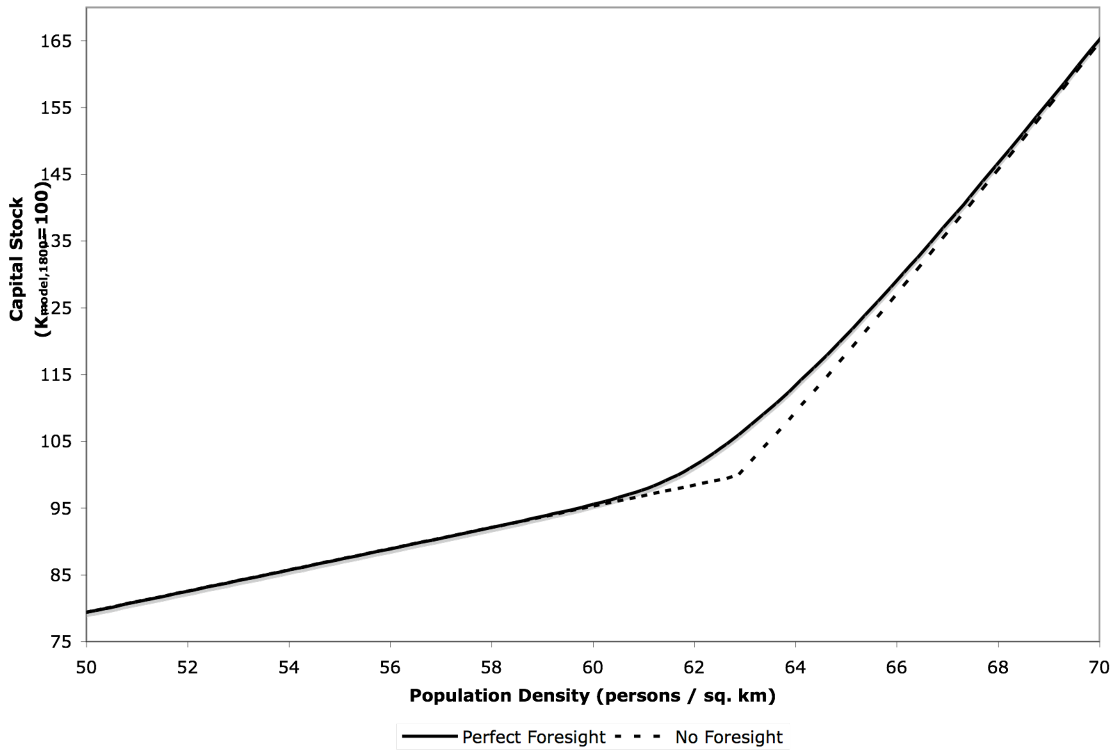


Table 3.1 compares this capital path to the data and finds that while the model accurately matches the capital growth immediately following industrialization, it underestimates capital growth both prior to industrialization and significantly following industrialization. The latter expected, since the calibration of the TFP growth path was chosen to simplify computation. The TFP path in Kahn (2007b), which accounts for more rapid growth in both sectors following industrialization, would more accurately match the data. However, the underestimation of capital growth prior to industrialization is not expected, especially since agents have perfect foresight. This finding may indicate that some additional aggregate forces played a role in causing industrialization.

Table 3.1: Growth Rate of Capital Stock, Great Britain[20]

Year	Capital Stock, Data (mil. GBP at 1851-60 prices)	Growth Rate of Capital Stock, Data	Real Captial Stock, Model (K_{1800}=100)	Growth Rate of Capital Stock, Model
1760	490	-	84	-
1800	730	49%	100	19%
1830	1180	62%	159	59%
1860	2310	96%	243	53%

Alternatively, this underestimation could be corrected if industrialization began slightly before 1800. As can be seen in Table 3.2, if the model is changed so that industrialization occurred in 1780, then the growth rate of capital from 1760-1800 increases to 49% without having a substantive effect on the other periods.

Table 3.2: Growth Rate of Capital Stock, 1780 Industrialization

Year	Capital Stock, Data (mil. GBP at 1851-60 prices)	Growth Rate of Capital Stock, Data	Real Captial Stock, Model (K_{1800}=100)	Growth Rate of Capital Stock, Model
1760	490	-	91	-
1800	730	49%	135	49%
1830	1180	62%	213	58%
1860	2310	96%	309	45%

Figure 3.10 plots the growth path of labor, broken down by sector, concurrently with the critical population density. Despite not being explicitly used in the solution algorithm, population density is seen to reach the critical population density at industrialization. Due to the manner in which it is calculated as a limiting condition, the critical population density is not well defined for $K_{a,t} \ll K_t$. Therefore, the critical population density is not depicted following industrialization.

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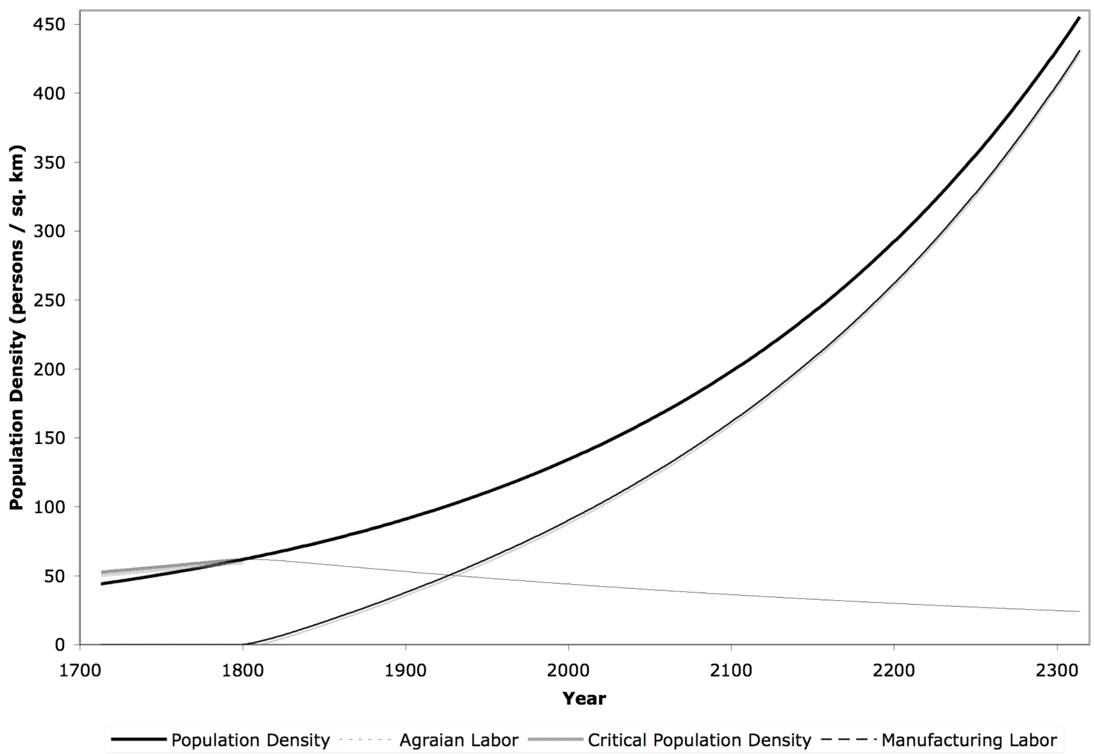
[20] Data obtained from Feinstein (1978). Defined as Gross Stock of Domestic Reproducible Fixed Capital, Great Britain.

As in data, Figure 3.2, the agrarian workforce increases prior to industrialization and then experiences a modest decline with the implementation of the manufacturing sector.

Figure 3.10: Labor Distribution and Critical Population Density



Also matching the data, in Figures 3.4 and 3.6 respectively, are the real rental rate of land, Figure 3.11, and the real wage, Figure 3.12.

Figure 3.11: Real Rental Rate of Land

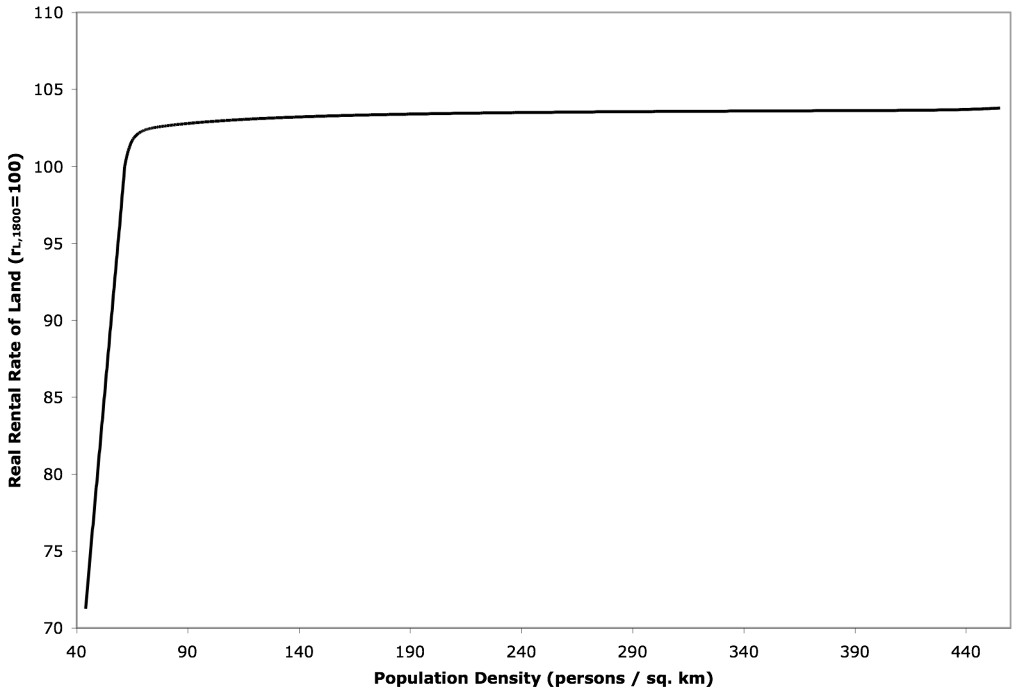
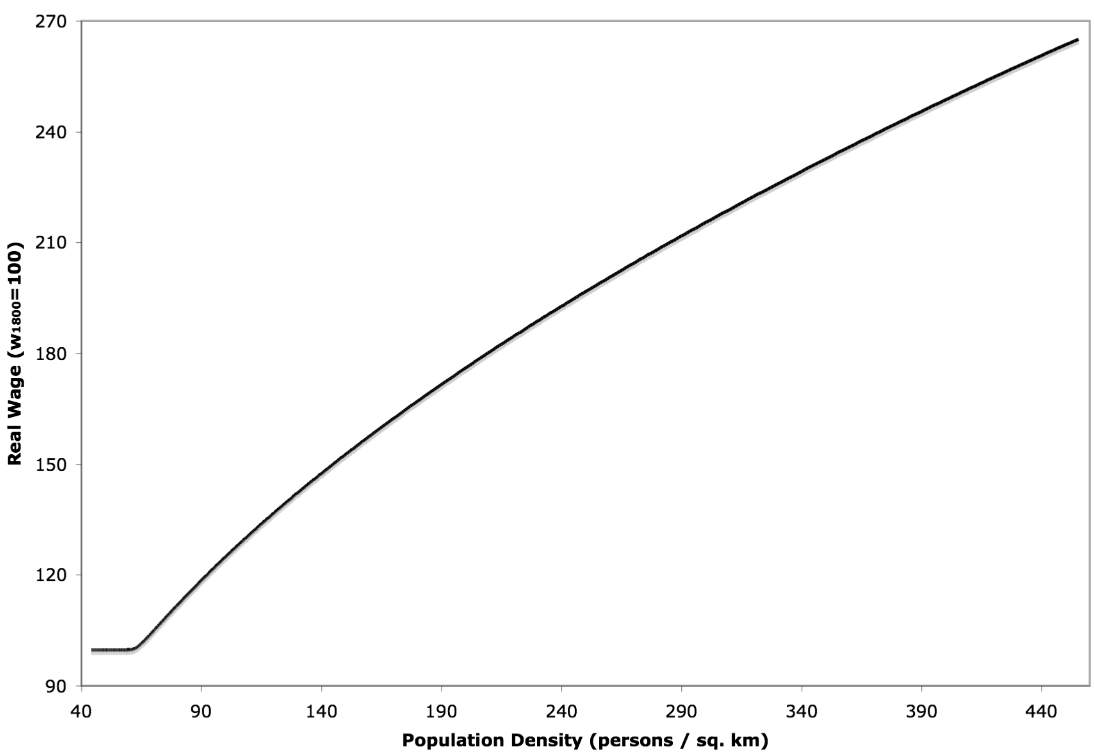
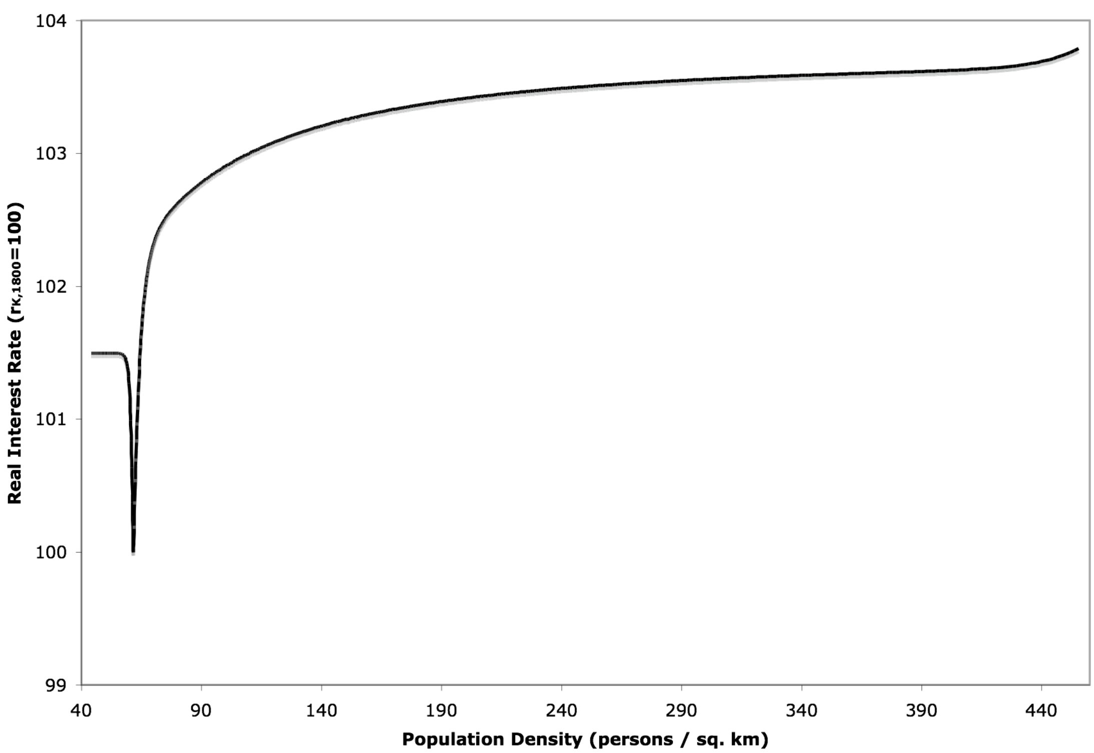


Figure 3.12: Real Wage



The return to capital, Figure 3.13, experiences the least movement of the factor prices, with the real return to capital settling at a level 4% higher after industrialization than before. This is consistent with the lack of long-term trend in real interest rates. The only abnormality is the slight drop in capital immediately prior to industrialization, which is a product of the increased rate of capital accumulation immediately prior to industrialization.

Figure 3.13: Real Return to Capital

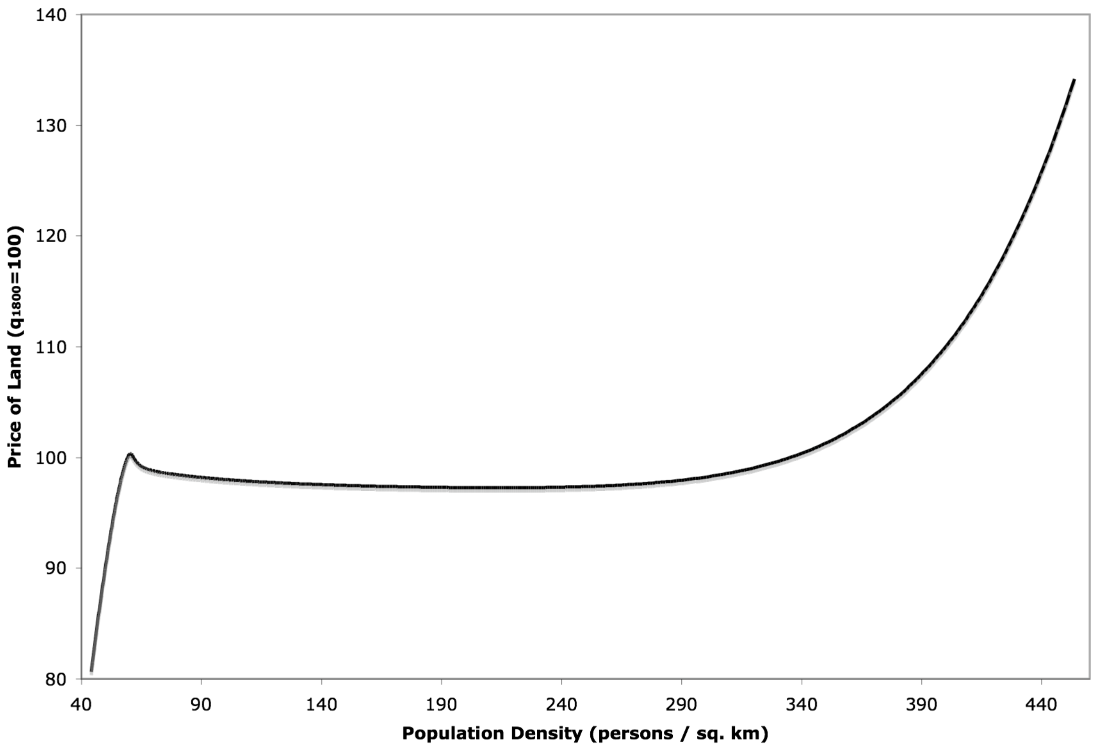


From Lemma 3 and the market clearing condition for land, it is know that $\lambda_{na,t} = \lambda_{L,t} = 0$ for all time. Therefore I divide (3.23) by (3.25) to obtain the sequence of land prices:

$$q_{t+1} = q_t \cdot \left(r_{Ka,t+1} + 1 - \delta\right) - r_{L,t+1} \tag{3.38}$$

Any sequence of land prices, $\{q_t\}_{t=1}^{\infty}$, that solves (3.38) for all t, will be viable for the equilibrium, and Figure 3.10 plots this sequence of land prices for the model.

Figure 3.14: Real Price of Land



VI. <u>Conclusion</u>

This paper has shown that in an infinite horizon framework, a perfectly competitive, Ricardian model with constant returns to scale agrarian and manufacturing technologies possess a critical population density upon which the manufacturing technology will be implemented.

Unlike in the case of an overlapping generations model, it is not possible to obtain a closed form solution for the critical population density. Instead, is an endogenous component exists. The other change that occurs with the shift to this framework is that agents adjust capital holdings in anticipation of industrialization. This manifests in an increase in capital per capita prior to the increase in output per capita that marks the onset of industrialization.

This increase in capital stock has the effect of hastening the onset of industrialization in comparison to a model in which agents lack foresight or are otherwise unable to deviate from the Malthusian steady state prior to the implementation of the industrial technology.

Despite this growth in capital per capita immediately prior to industrialization, the model still underestimates the capital growth seen in the data during this period, but matches it accurately immediately following industrialization. While there are many such explanations for this underestimation, one way to correct for this difference is to set 1780 as the starting date of industrialization, rather than 1800. With this new starting date, the growth rate of capital predicted by the model misses the data by only 0.4% in the 30 years preceding industrialization and 5.5% in the 30 years following industrialization.

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